Modelling the Volatility of the Spanish Wholesale Electricity Spot Market. Asymmetric GARCH Models vs. Threshold ARSV model

JOSÉ M. MONTERO
Departamento de Estadística, UNIVERSITY OF CASTLLA-LA MANCHA, SPAIN. E-mail: Jose.mlorenzo@uclm.es

MARIA C. GARCÍA-CENTENO
Departamento de Estadística, UNIVERSITY OF SAN PABLO-CEU, SPAIN. E-mail: garcen@ceu.es

GEMA FERNÁNDEZ-AVILÉS
Departamento de Estadística, UNIVERSITY OF CASTLLA-LA MANCHA, SPAIN. E-mail: gema.faviles@uclm.es

ABSTRACT

The liberalization and deregulation of the Spanish electricity market has provoked an increase in the complexity of pricing behaviour. In particular, the volatility of electricity spot prices is the feature that best characterises the current Spanish market. Since an understanding of the volatility process in the electricity market is critically important to distributors, generators and market regulators, this article focuses on the asymmetrical pattern of the volatility of Spanish electricity spot prices, paying special attention to the direct or inverse leverage effect. For this purpose, we use both a range of traditional GARCH models and a T-ARSV model. The results clearly favour the proposed T-ARSV specification, which suggests a positive leverage effect in the Spanish market.

Keywords: Electricity Prices, Volatility, GARCH, T-ARSV.

Modelización de la Volatilidad en el Mercado Eléctrico Español. Modelos GARCH frente al modelo T-ARSV

RESUMEN

El proceso de liberalización y desregulación del mercado eléctrico español ha incrementado la complejidad del comportamiento de los precios. En particular, la volatilidad de los precios spot es la característica que mejor define el mercado español actual. Teniendo en cuenta que el conocimiento de este hecho estilizado es clave para distribuidores, generadores y reguladores, en este artículo nos centramos en el estudio de la respuesta asimétrica o no de los precios spot, así como en la existencia de efecto leverage directo o inverso. Para ello se utiliza una batería de modelos GARCH tradicionales en la literatura, a la que se enfrenta el modelo de volatilidad T-ARSV. Los resultados favorecen a la especificación T-ARSV y sugieren un efecto leverage positivo en el mercado eléctrico español.

Palabras clave: Precios de electricidad, volatilidad, GARCH, T-ARSV.

1. INTRODUCTION

The two last decades have witnessed a liberalization and deregulation of electrical monopolies worldwide. This process of liberalization and deregulation has provoked an increase in the complexity of pricing behaviour. Thus, there is an increasing need to understand how the planning methods used under monopoly will have to change in order to take the new deregulated environment into account. Decision making in these deregulated markets is extremely difficult because the markets are very complex and highly changeable. It is therefore unsurprising that risk management constitutes an issue of increasing importance, playing a core role in these new deregulated markets.

Understanding the volatility process in the electricity market is critically important to distributors, generators and market regulators, as it influences the pricing of derivative contracts traded on electric power prices, therefore allowing them to better manage their financial risks. As stated in Gianfreda (2010) and the references therein, it also is crucial for firms to formulate their supply schedules and evaluate real investments using modern asset pricing techniques, and for consumers to hedge against the risk of price spikes or the risks associated with physical delivery. Additionally (Thomas and Mitchell, 2005), system operators and industry regulators also need to understand volatility to ensure that markets are designed and operated in a way that limits market power and promotes confidence and safety for market participants.

In this article, we focus on the asymmetric pattern of the volatility of electricity spot prices. This aspect has not been sufficiently studied in the existing literature, especially within the Spanish wholesale market, but is extremely important when it comes to understanding the volatile behaviour of electricity prices. It should be taken into account that if the economic consequences of volatility are important and destructive, these consequences intensify if this volatility has an asymmetrical pattern, whose direction seems to depend on the markets in question. This therefore is the reason that we will pay special attention to this core stylized fact. In addition, the analysis of the asymmetrical pattern of volatility is critical because the existing literature draws no conclusions about the existence of a direct or inverse leverage effect.

Competitive electricity markets share high volatility -as well as assorted other important features including high-frequency trading, non-constant mean and variance prices, multiple seasonality, the weekend and holiday effect, a significant number of 'outlier' prices and dependency on explanatory variables- with financial markets. But, volatility in electricity markets is even higher than in financial markets, and in some markets its asymmetric pattern is suspected to be opposite to that of the financial returns (the above-mentioned *inverse leverage effect*). This is the reason why volatility is considered to be the feature that best characterises the competitive electricity markets.

Since the liberalization and deregulation of electricity markets is a recent event, research about electricity pricing and on the volatility of electricity prices in particular is relatively new, but vast and diverse. Restricting ourselves to the recent literature, which focuses on the asymmetric response of volatility in wholesale electricity markets, GARCH-type models after filtering outliers are the common instrument used to analyse the asymmetrical pattern of volatility. Without wishing to be exhaustive, after the works of Clewlow and Strickland (2000), Weron *et al.* (2001), Huisman and Mahieu (2001), Lucia and Schwartz (2002) and Weron *et al.* (2004), in the framework of the econometric approach based on discrete time, Higgs and Worthington (2004) investigate the intraday price volatility process in four Australian wholesale electricity markets using a range of processes including GARCH, Risk Metrics, normal Asymmetric Power ARCH (APARCH), Student APARCH and skewed Student APARCH. Hadsell *et al.* (2004) examine the volatility of the wholesale electricity prices for five US markets using TARCH models. Biström (2005) studies the changes in Norwegian electricity prices combining an AR-GARCH model with the extreme value theory. Hua *et al.* (2006) also use GARCH for volatility modelling. Chan and Gray (2006) use the EGARCH modelling for the analysis of returns. Thomas and Mitchell (2005) consider the underlying volatility process in five regional pool markets in the NEM (Australia), and examine the applicability of a range of GARCH specifications, including the basic GARCH, TGARCH, EGARCH and PARCH models. Kâ Diongue and Guedan (2008) propose what they call the GGk-APARCH (k-factor Gegenbauer with Asymmetry Power GARCH) process for modelling the leverage effect and other stylized facts of electricity prices. Naeem (2010) compares ARMA-GARCH models and mean-reverting Ornstein-Uhlenbeck models for their respective capability to capture the statistical properties of real electricity spot market time series. His conclusion is that neither ARMA-GARCH models nor conventional mean-reverting Ornstein-Uhlenbeck models capture the statistical characteristics of the real series. Gianfreda (2010) analyses the volatility of wholesale electricity markets for five markets in Europe using GARCH(1,1) and EGARCH(1,1) models after filtering the anomalous values.

In this paper, four commonly used GARCH-type models are estimated to compete with the threshold autoregressive stochastic volatility (T-ARSV) model to explain the asymmetric pattern of volatility. The GARCH models include: (i) the AGARCH model proposed by Engle (1990), (ii) the threshold generalized autoregressive heteroskedasticity (TGARCH) model, proposed by Zakoian (1990) (iii) the EGARCH model developed by Nelson (1991) and the GJR model proposed by and Glosten *et al.* (1993). The threshold autoregressive stochastic volatility (T-ARSV) model was proposed by So *et al.* (2002). However, for estimation purposes we follow García and Mínguez (2009). We

also estimate the ARSV model because it is nested in T-ARSV. We use this model to test the symmetric response hypothesis.

As far as we know, the T-ARSV model has never been used to describe the volatility pattern of electricity prices. However, it has been successfully used to describe the volatility behaviour of returns of other energy products, including crude oil (OPEC reference basket and London Brent index), unleaded regular oxygenated or reformulated petrol, natural gas, butane and propane (Montero *et al.*, 2010).

The A-ARSV model proposed by Harvey and Shephard (1996) and developed in Yu (2005), Asai and McAleer (2006), Ruiz and Veiga (2008b) and Smith (2009), is another well known stochastic volatility model for coping with the leverage effect including the correlation between both disturbances in the model, that is to say $E\left(\varepsilon_t \eta_{t+1}\right) = \delta \sigma_\eta$. However we do not estimate it because this article focuses on the analysis of the asymmetric response of volatility with threshold models.

We have focussed our attention in the Spanish electricity spot market because although most of the existing literature refers to multiple pools in Australia, the U.S. and the Nordic Power Exchange ("NordPool"), the Spanish case has not received sufficient attention (the details of trading and the rules are complex, and given space constraints this article does not go into details. See León and Rubia, 2001, and Alonso, 2008, for a fuller explanation). We focus on the returns of the spot (in fact a Day-Ahead) market. The Day-Ahead markets have a high degree of volatility, because both supply and demand depend on a large number of variables, including supply factors (prevision of wind energy production, price of fuel and CO_2, hydraulic reserves, availability of power stations, etc.) and demand factors (labour factors, temperature, etc.). Forward markets are used to avoid the high degree of volatility of the Day-Ahead markets, but they have an additional risk component: the credit risk. They allow the participants in the market to have a larger or lesser exposure to risk according to their financial strategy. In brief, forward markets can be seen as a solution for agents that need to manage their price or volume risk. Forward contracts must reflect the future expectative of the spot Day-Ahead markets and the forward price varies according to such an expectative. It therefore fluctuates according to the agent's expectative and uncertainty about the factors that affect the spot price. Thus, the adequate modelling of volatility in the spot market allows for a more accurate estimation of the margins needed when negotiating contracts in the forward and derivatives markets.

The time reference we consider is January 2000–October, 2010, probably the largest and most recent interval studied in the literature.

The remainder of the paper is organized as follows: After this introduction, section 2 is devoted to methodological aspects. Section 3 delineates the main features of electricity returns in the Spanish spot market and reports the empirical results derived from the comparison of T-ARSV and traditional GARCH models. Finally, Section 4 concludes the paper.

2. METODOLOGICAL ASPECTS

We propose a T-ARSV model to explain the dynamics of the volatility of electricity returns. We also estimate the symmetric ARSV model because it is nested in the T-ARSV specification, and use it to test whether or not the parameters indicating the asymmetric response of volatility in the T-ARSV model are significantly equal. This model is compared with the AGARCH, TGARCH, EGARCH and GJR models, which are the most common GARCH-based models in the literature (see Rodriguez and Ruiz, 2009, for details on the statistical properties of some of the most popular GARCH models with leverage effect when their parameters satisfy the positivity, stationarity and finite fourth order moment restrictions). All these models have the same mean equation:

$$y_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim i.i.d \ N(0,1) \tag{1}$$

where y_t are the returns, σ_t^2 represents the conditional variance, and ε_t, the innovations, are independent and follow a Gaussian distribution with zero mean and unit variance. The models differ in the conditional variance specification as indicated in Table 1. We propose to explain the dynamics of returns volatility using AGARCH(1,1), TGARCH(1,1), EGARCH(1,1), GJR(1,1), ARSV(1), and T-ARSV(1). The specification of the conditional variance in the abovementioned models is reported in Table 1.

Table 1
Models for the specification of the conditional variance

Models	Conditional variance equation
AGARCH(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 (y_{t-1} - \gamma)^2 + \beta \sigma_{t-1}^2$
TGARCH(1,1)	$\sigma_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma \varepsilon_{t-1} d_{t-1} + \beta \sigma_{t-1}$
EGARCH(1,1)	$\log \sigma_t^2 = \omega + \beta_1 \log \sigma_{t-1}^2 + \alpha \dfrac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}}} - \alpha \sqrt{\dfrac{2}{\pi}} + \gamma \dfrac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}}$
GJR(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2$
ARSV(1)	$h_t = \phi h_{t-1} + \eta_t$
T-ARSV(1)	$h_t = (\phi_{11} I_{1t} + \phi_{12} I_{2t}) h_{t-1} + \eta_t$

Source: Own elaboration.

As is well known, in the AGARCH(1,1) model, γ is the parameter that accounts for the asymmetrical behaviour of volatility, and the asymmetric effects (if $\gamma \neq 0$ and significant) are captured by $\alpha_1(y_{t-1} - \gamma)^2$. The TGARCH and GJR models incorporate an indicator variable, $d_{t-1} = \begin{cases} 1 & \varepsilon_{t-1} < 0 \\ 0 & \varepsilon_{t-1} \geq 0 \end{cases}$, representative of the good news ($\varepsilon_{t-1} \geq 0$) and the bad news ($\varepsilon_{t-1} < 0$) in the market. Thus, in case of good news the effect on volatility is α_1 and in case of bad news the impact is $\alpha_1 + \gamma$. In these two specifications, α_0 is positive and both α_1 and β are nonnegative (to guarantee a nonnegative variance). In addition, following the previous GARCH specification, the condition $\alpha_1 + \beta < 1$ is satisfied to ensure the stationarity of the process y_t.

The difference between TGARCH and GJR is that TGARCH models the conditional standard deviation, while GJR models the conditional variance. Note in Table 1 that the conditional variance equation in the EGARCH(1,1) model has been expressed, as usual, in logarithmic form. This is a way to avoid imposing the parametric restrictions $\alpha_0 > 0, \alpha_1 \geq 0, \beta \geq 0$. In this specification $(\alpha + \gamma)$ is the effect on volatility of a previous positive return, whereas $(\gamma - \alpha)$ is the corresponding effect when $y_{t-1} < 0$. In the ARSV(1) model, $\sigma_t = \sigma_* \exp(0.5h_t)$, where h_t is the log-volatility, $h_t = \log \sigma_t^2 - \log \sigma_*^2$, and σ_* is a positive scale factor in the mean equation to avoid including a constant in the log-volatility equation. η_t is a white noise process, and it follows a Gaussian distribution with zero mean and variance σ_η^2, and both ε_t and η_t are independent, $E(\varepsilon_t, \eta_t) = 0, \ \forall t, s$. This way of specifying the conditional variance guarantees the positivity of the variance. For more details about these GARCH models, see Laurent (2007).

The T-ARSV(1) model is the other model proposed to describe the dynamics of the volatility. The mean equation is defined as in the ARSV(1) model, Eq. (2), but the way of specifying the conditional variance is different.

The asymmetrical pattern of volatility rests on establishing a known threshold that changes the value of the parameters in the model. Therefore, obtaining the T-ARSV model from an ARSV model implies:

   a) adding two new parameters, ϕ_{11} and ϕ_{12}, which measure the effect of the positive and negative returns on volatility, respectively.

   b) adding two indicator variables, I_{1t} and I_{2t}, defined as:

$$I_{1t} = \begin{cases} 1 & \text{when the price variation is zero or positive} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{2t} = \begin{cases} 1 & \text{when the price variation is negative} \\ 0 & \text{otherwise} \end{cases}$$

Therefore, the T-ARSV(1) model can be seen as a generalisation of the ARSV(1) model which includes two additional parameters that allow for an asymmetrical pattern of volatility. Volatility is defined as an exponential function. Thus, the model is not linear. However, for estimation purposes it can be expressed in a linear form by squaring the mean equation and taking logarithms. Following Sandmann and Koopman (1998), we express the specification in a space state form to obtain the following linear model:

$$\begin{pmatrix} h_{t+1} \\ Y_t \end{pmatrix} = \delta_t + \mathbf{\Phi} \mathrm{h}_t + u_t \tag{2}$$

where $Y_t = \log y_t^2$ and

$$\mathbf{u}_t \sim i.i.d. \ N(\mathbf{0}, \mathbf{\Omega}_T), \quad \mathbf{\delta}_t = \begin{pmatrix} 0 \\ \ln \sigma_*^2 \end{pmatrix}, \quad \mathbf{\Phi} = \begin{pmatrix} \phi_{11}\mathbf{I}_{1t} + \phi_{12}\mathbf{I}_{2t} \\ 1 \end{pmatrix}, \quad \mathbf{\Omega}_T = \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \dfrac{\pi^2}{2} \end{pmatrix} \tag{3}$$

The unknown likelihood function of a T-ARSV(1) model, which is a non-Gaussian model, has been evaluated by using the Monte Carlo method, approximating the non-Gaussian model by importance sampling (Durbin and Koopman, 1997, Sandmann and Koopman, 1998).

The estimation of the parametric vector $\left(\phi_{11}, \phi_{12}, \sigma_*, \sigma_\eta^2\right)$ requires the following algorithm:

a) An approximated Gaussian model is obtained from an initial vector of parameters of the model. The initial values of the parameters are estimated from the available information for each return.

b) The Gaussian likelihood function for the approximated model is calculated using the Kalman filter.

c) The process is repeated until the desired level of convergence is achieved. At this level, the likelihood function reaches its maximum value.

Finally, the parametric vector that maximizes the simulated likelihood function is obtained by using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) method, a well-known method to solve unconstrained nonlinear optimization problems.

The leverage effect is checked by testing the null hypothesis: $H_0 : (\phi_{11} = \phi_{12})$ (ARSV model) versus the alternative one: $H_1 : (\phi_{11} \neq \phi_{12})$ or T-ARSV(1) model. Since the null and the alternative hypotheses refer to two nested models, this strategy allows for the implementation of a likelihood ratio test, the test statistic being $\lambda = -2(\log L^R - \log L)$, which follows (under the null) a chi-squared distribution with one degree of freedom.

If the null hypothesis is not rejected, then there is no evidence of an asymmetric pattern of volatility. In this case, the ARSV(1) model could be preferred. On the other hand, the rejection of the null hypothesis suggests that the effects of positive and negative shocks on the dynamics of the volatility are different.

The estimation procedure for ARSV(1) model has been conducted in Ox language and it is available at www.feweb.vv.nl/koopman/sv. The estimation of T-ARSV(1) has been carried out with a new proprietary code using Ox 4.1. language following the steps proposed by Shephard and Pitt (1997), Durbin and Koopman (1997) and Koopman and Hol (2002) for ARSV(1) modelling.

3. EMPIRICAL APPLICATION: SPANISH ELECTRICITY SPOT RETURNS

As in Lucia and Schwartz (2002), Worthington *et al.* (2005) and Gianfreda (2010), amongst others, we deal with the daily marginal prices reported by OMEL (the economic management of the electricity market in Spain is entrusted to OPERADOR DEL MERCADO IBÉRICO DE ENERGÍA – POLO ESPAÑOL, S.A, http://www.omel.es/files/flash/ResultadosMercado.swf.). As stated in Worthington *et al*. (2005), this treatment will entail the loss of at least some 'news' impounded in more frequent trading interval data but this drawback is widely compensated by the fact that daily averages play a crucial role in electricity markets, particularly in the case of financial contracts. As an example, MIBEL PTEL Base Load Financial Futures Contracts have as the spot reference price the monetary value of the PTEL Base Index (1€/index point), which is equivalent to the arithmetic mean of all the Spanish hourly marginal prices of OMEL's Daily Market, for the Portuguese system. The spot reference price for MIBEL SPEL Base Load Financial Futures Contracts is the monetary value of the SPEL Base Index (1€/index point), which is equivalent to the arithmetic mean of all the Spanish hourly marginal prices of OMEL's Daily Market, for the Spanish system. The spot reference price for both Base Load SPEL Forward Contracts and Base Load SPEL Swap Contracts, for each delivery day, is the monetary value of the SPEL Base Index (1€/index point), which is equivalent to the arithmetic mean of all the Spanish hourly marginal prices of OMEL's Daily Market, for the Spanish system.

Some authors (Thomas and Mitchell, 2005, and Thomas *et al*., 2006, among others) use hourly or half-hourly trading-interval prices to allow for the presence of negative prices but in the Spanish market they are not a significant feature of the data.

The daily series (January 1, 2000, to October 26, 2010) has been obtained from the series of hourly prices and is shown in Figure 1. As in Thomas and Mitchell (2005), we believe the use of a very-much-larger data set than that which is usually studied better characterises the volatility process by examining the market over a wider range of conditions and a broader market base.

In Figure 1 the high volatility of Spanish electricity spot prices appears evident. By comparing volatility in the Spanish electricity spot market with that observed in IBEX-35 (the main index of Madrid Stock Exchange), it can be noticed that the former is 50% and the latter 38% during the period under study. This is not an idiosyncratic feature of Spanish markets (see Weron, 2006, for comparisons with other stocks, crude oil and natural gas). What is more, in other markets (United States, Australia, Germany, France, among others), the volatility of electricity prices is similar (around 50%) but the volatility of stock prices is substantially lower.

Figure 1
Dynamics of average daily electricity spot prices in Spain: 1/1/2000–26/11/2010
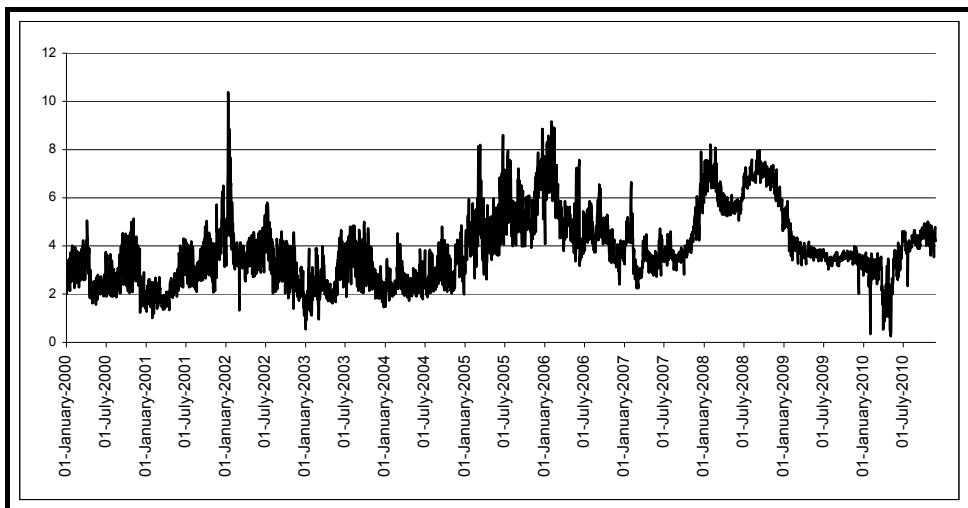
Note: Figure 1, illustrates that:
    (i) Prices are not stationary, neither in mean nor in variance.
    (ii) Some extreme spikes can be observed.

Source: OMEL.

The existence of spikes[1] is one of the factors that make the modelling of electricity prices most difficult. The literature offers two options for coping with this problem: (i) the normal regime, where the anomalous values are filtered, and (ii) the abnormal regime, where spikes are taken into account. Researchers on the topic usually filter the anomalous values and focus on the normal regime. However, we favour the other option because: (i) since the number of spikes is usually large, the number of observations could be substantially reduced, (ii) to arbitrarily reduce the spiky magnitude is only a second and arbitrary best, and (iii) we believe that an appropriate strategy to model volatility should be robust enough to take spikes into account.

After deciding to focus on the abnormal regime, since prices are non-stationary, we deal with logarithms to correct the dispersion problem. This logarithmic specification also dampens the effects of extreme spikes. To correct from trend, we use a first-difference (Δ). To overcome the weekend effect we calculate a weekly lag (Δ_7). These transformed prices ($\Delta_7 \Delta logPrice$) are what we call *returns*. As stated in Black (1976), we are aware that there is no ability to hold a unit of electricity and that there is no initial investment in the commodity as such. Thus, spot electricity does not yield a return to an investor in the traditional sense. However, this terminology is widely used in the literature.

Figure 2 illustrates that electricity spot returns have a null mean (see Table 1) but non-constant variance. Specifically, some volatility clusters can be detected, which can be indicative of a non-constant conditional variance. Electricity returns are also uncorrelated ―see Figure 3; in addition, the Box-Ljung statistic is $Q_{y_t}(20) = 12.6588$, which indicates that the correlation coefficients are insignificant at a 0.05 significance level― although they are not independent, because squared returns are positive and significantly correlated (Figure 4). In addition, the value of the Box-Ljung statistic for the corresponding series is $Q_{y_t^2}(40) = 1404.2$, which is significant at the 0.05 significance level; thus, there exist dependence on the squared returns[2]. Marginal distributions of returns are both asymmetric and leptokurtic (Table 2). Figure 5 shows, in the upper panel, the cross-correlations between returns (y_t)

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[1] As stated in Blanco and Soronow (2001) and reproduced in Thomas and Mitchell (2005), we prefer the term 'spike' to 'jump', because a jump process in financial markets usually suggests that the prices move rapidly to a new level and remain there. However, electricity prices tend to move abruptly to an extremely high level and revert to mean just as abruptly.

[2] Figure 3 suggests persistence of volatility. This is not the topic of the article, but details on asymmetric long memory volatility models as FIEGARCH or A-LMSV can be found in Bollerslev and Mikkelsen, 1996, and Ruiz and Veiga, 2008a, respectively.

and the absolute value of such returns ($|y_{t+k}|$); the lower panel reports the cross-correlations between returns and squared returns (y_t^2). It can be appreciated that both cross-correlations are significant, which could indicate the existence of leverage effect[3] (see Ruiz and Veiga, 2008a, 2008b).

Table 2
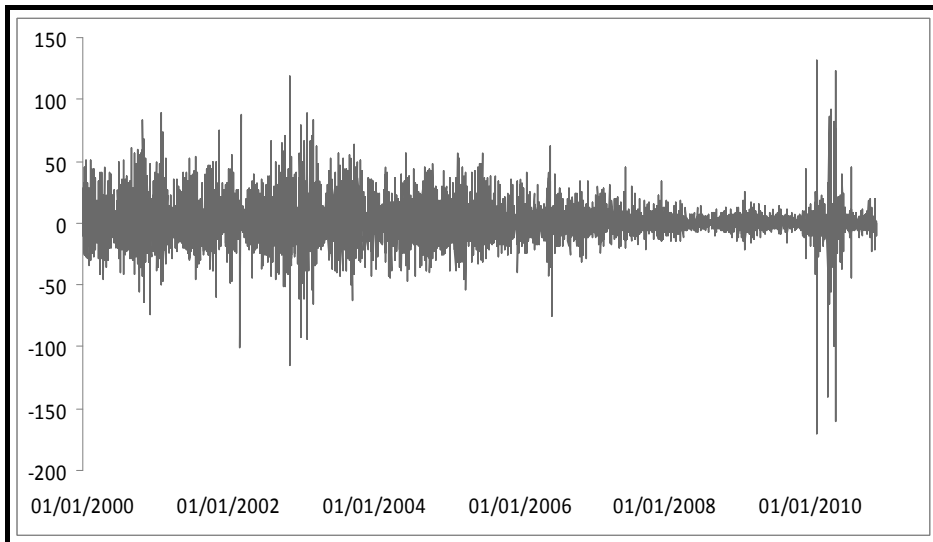Electricity returns (Spanish spot market): Basic statistics

Mean	Standard deviation	Skewness coefficient	Kurtosis coefficient	J-B Normality test
0.019185*	8.540	0.14073	8.5156	3243.1**

* Insignificant at the 5% level.
** Significant at the 5% level.
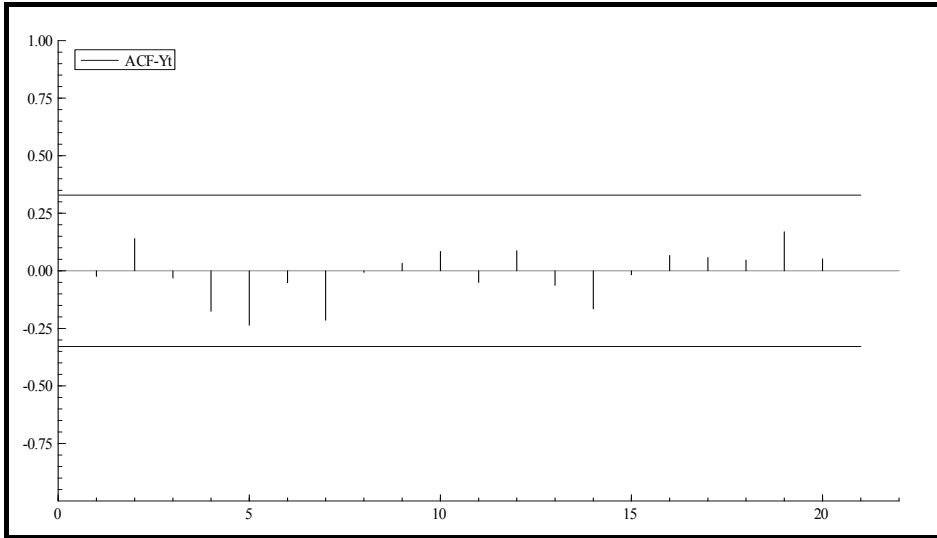*Source:* Own elaboration based on data from OMEL.

Figure 2
Electricity returns in the Spanish spot market (1/1/2000−26/11/2010)

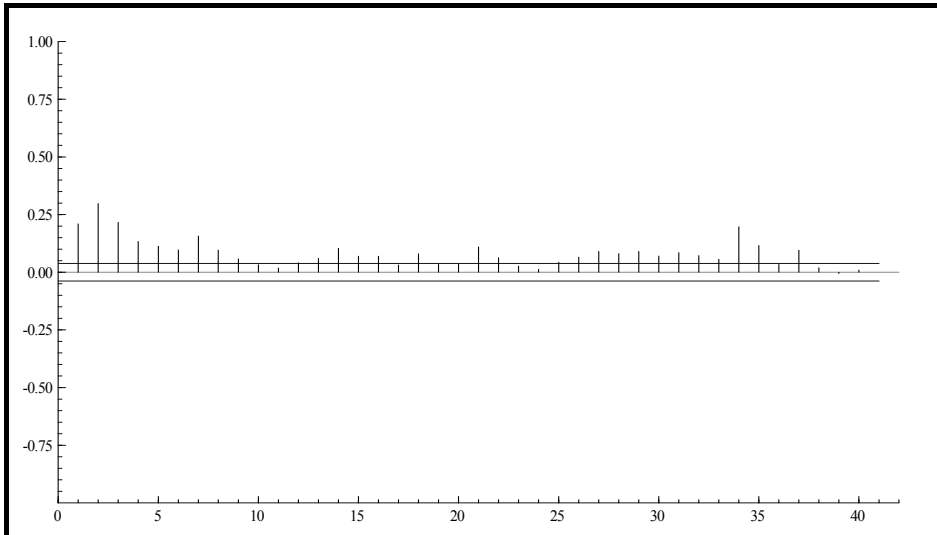*Source:* Own elaboration based on data from OMEL.

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[3] The asymmetry observed in financial data was initially attributed to the effects of leverage, but this could not be the case in other markets. In any case we will use the term 'leverage effect'.
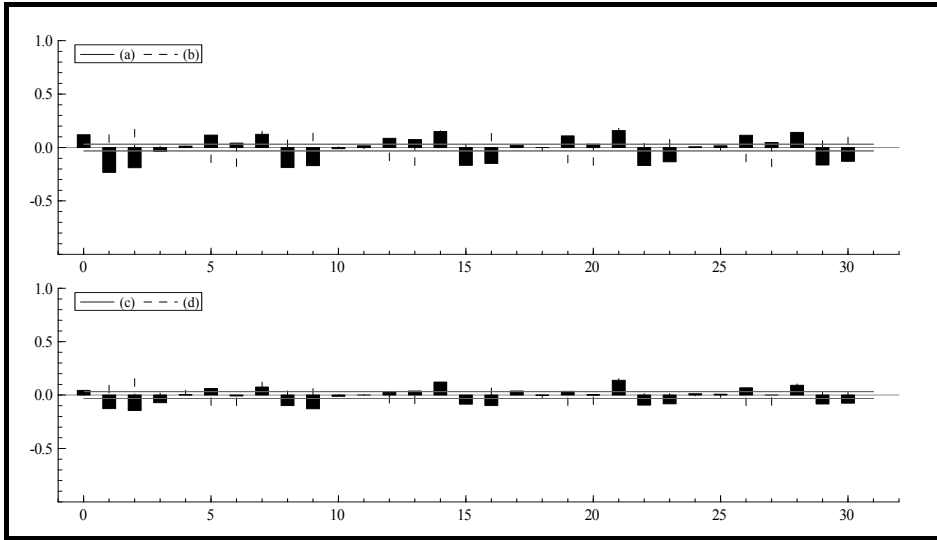
Figure 3
ACF of returns

Source: Own elaboration based on data from OMEL.

Figure 4
Autocorrelation function (ACF) of squared returns

Source: Own elaboration based on data from OMEL.

Figure 5
Cross-correlation functions

(a) y_t and ; $|y_{t+k}|$; (b) $|y_{t+k}|$ and y_t ; (c) y_t and y_t^2 ; (d) y_t^2 and y_t

Source: Own elaboration based on data from OMEL.

In addition, there is a weekend effect:

$$\log price = \underset{(0.00115)}{1.182} + \underset{(0.0136)}{0.168} \, Dummy(week\ day - weekend) \tag{4}$$

The above-mentioned features of the volatility of Spanish daily spot returns suggest that it can be modeled. GARCH models are the specifications traditionally used in the literature to estimate the volatility of electricity returns, but we propose the T-ARSV model, which is more powerful than GARCH models when it comes to capturating the stylised facts of electricity returns, especially the volatility pattern, the stylised fact that we focus on in this work.

For clarity, empirical results are reported in two tables. Table 3 reports the results obtained from the estimation of the AGARCH, TGARCH, EGARCH and GJR models. Table 4 presents the results for ARSV and T-ARSV models. And Table 5 focus on the stability of T-ARSV model. As stated above, we focus our attention on one of the most important stylized facts of the Spanish electricity spot market: the existence of leverage effect (direct or inverse).

As demonstrated in Table 3, γ, the parameter that indicates the presence of leverage effects, is positive in all GARCH models except in the EGARCH specification, but is only significant in the AGARCH(1,1) and EGARCH(1,1) models. According to the specification for AGARCH(1,1) reported in Table 2

(note that γ is preceded by a negative sign), this indicates that returns in $t+1$ are more volatile in the case of a negative return in period t than in the case of a positive return. As for the EGARCH(1,1) model, note that magnitude effect is measured by $\alpha \dfrac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}}} - \alpha \sqrt{\dfrac{2}{\pi}}$ and the sing effect is indicated by $\gamma \dfrac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}}$. Since the γ parameter is negative, there is a strong market response to negative news. The TGARCH(1,1) and GJR(1,1) models do not detect an asymmetrical pattern of the Spanish daily spot electricity prices. That is to say, insignificant asymmetric volatility coefficient was found for both specifications, which suggests the same reaction pattern to both positive and negative shocks.

The T-ARSV model also detects the asymmetrical pattern of volatility in returns (Table 4), and in the same direction as AGARCH (1,1), since ϕ_{12} exceeds ϕ_{11}. This fact implies that in case of an increase in electricity spot prices, the volatility, and hence the uncertainty, is greater than in case of a decrease in price. As a consequence, the T-ARSV(1) model could also be an appropriate candidate for modelling the behaviour of the volatility of electricity spot returns.

Therefore, the traditional AGARCH(1,1) and EGARCH(1,1) models obtain the same result as the stochastic volatility T-ARSV(1) model. However, the AGARCH(1,1) model cannot be considered a good choice to estimate the volatility of the electricity returns in the Spanish spot market because the sum of the α and β values is the unity in the AGARCH(1,1,) model suggesting an explosive process and a potentially unstable model.

Ranking by Akaike Information (AIC), Hannan-Quinn (HQC) and Schwartz Bayesian Criteria favours the T-ARSV model over the EGARCH(1,1), see Table 6.

Table 3
Estimates of Asymmetric GARCH(1,1)-Type models coefficients

	AGARCH	TGARCH	GJR	EGARCH
α_0	-0.0056 (-1.93)	0.0001 (1.10)	0.0000 (1.02)	-0.0120 (-7.891)
α_1	0.1576 (2.03)	-0.0032 (-0.150)	-0.0037 (-0.175)	-0.4312 (-5.566)
β	0.8423 (13.6)	0.8776 (12.8)	0.8830 (13.3)	0.9702 (127.0)
γ	0.04755 (4.66)	0.3105 (1.65)	0.3125 (1.63)	-0.4726 (-16.360)
α	—	—	—	0.2548 (7.304)
$\alpha_1 + \beta$	1	0.8744	0.8793	0.9702

The values between parentheses indicate the value or the t-statistic.
Source: Own elaboration based on data from OMEL.

Table 4
Estimates of ARSV(1) and T-ARSV(1) models

T-ARSV				ARSV			LR[1]
σ_*	ϕ_{11}	ϕ_{12}	σ_η^2	σ_*	ϕ	σ_η^2	λ
0.049 (0.085)	0.962 (0.409)	0.999 (0.344)	0.012	0.042 (0.084)	0.986 (0.252)	0.017	10.6

[1]Likelihood Ratio Test (LR). $\lambda = -2(\ln L^R - \ln L)$. Critical value: 3.84 (5%).

The values between parentheses indicate the estimated standard deviation.

Source: Own elaboration based on data from OMEL.

Table 5
Stability of T-ARSV model

T-ARSV					
	Sample size	σ^*	ϕ_{11}	ϕ_{12}	σ_η^2
2004-2010	2522	0.060 (0.093)	0.954 (0.412)	0.998 (0.416)	0.007
2003-2010	2887	0.058 (0.091)	0.958 (0.432)	0.998 (0.470)	0.010
2002-2010	3252	0.062 (0.087)	0.958 (0.403))	0.998 (0.545)	0.010
2001-2010	3617	0.062 (0.089)	0.955 (0.382)	0.999 (0.403)	0.011
2000-2010	3975	0.049 (0.085)	0.962 (0.409)	0.999 (0.344)	0.012

The values between parentheses indicate the estimated standard deviation.

Source: Own elaboration based on data from OMEL.

Table 6
Information criteria for Asymmetric GARCH and T-ARSV models

	AIC	HQ	Schwartz
AGARCH	-1,0131	-1,01426	-1,01531
TGARCH	-1,0547	-1,05585	-1,05691
GJRGARCH	-1,0551	-1,05673	-1,05778
EGARCH	-1,1043	-1,10548	-1,10654
T-ARSV	-1,1184	-1,11957	-1,12062

Source: Own elaboration based on data from OMEL.

Our findings contradict the finding of Knittel and Roberts (2005), who advocate for the existence of an inverse and perverse leverage effect (a positive shock will increase the volatility more than a negative one). On the contrary, our findings confirm the results of Gianfreda (2010) for Spanish, German, French and Italian markets using an EGARCH model, the finding previously obtained by Petrella and Sapio (2009) on the Italian PUN prices, and the results of most of the Australian GARCH-based studies.

4. CONCLUSIONS

Volatility in electricity markets is even higher than in financial markets and in some markets its asymmetric pattern is suspected to be opposite to that of the financial returns (the well-known *inverse leverage effect*). Therefore, modelling the volatility behaviour, checking whether its response is symmetrical or not and detecting the presence or not of the perverse leverage effect are still core open questions in this topic, especially in the Spanish market due to the scarceness of studies in the literature.

In this article, we propose a threshold autoregressive stochastic volatility (T-ARSV) specification to compete with four commonly used GARCH-type models (AGARCH, TGARCH, EGARCH and GJR). We also estimate the ARSV model, because it is nested in T-ARSV and we use it to test the symmetric response hypothesis. As far as we know, the T-ARSV model has never been used to describe the volatility pattern of electricity prices. However, it has been successfully used to describe the volatility behaviour of returns of other energy products that include crude oil (OPEC reference basket and London Brent index), unleaded regular oxygenated or reformulated petrol, natural gas, butane and propane.

Results obtained for the Spanish wholesale electricity spot markets indicate that the T-ARSV model detects the asymmetrical pattern of volatility in returns. This asymmetrical response is also detected by AGARCH (1,1) and EGARCH(1,1) models. However, the AGARCH(1,1) model is a potentially unstable model and Akaike Information (AIC), Hannan-Quinn (HQC) and Schwartz Bayesian Criteria favours the T-ARSV model over the EGARCH(1,1) model.

Another core finding is that, unlike in other electricity markets, the existence of an inverse leverage effect can be rejected in the Spanish electricity market. That is to say, as in the stock markets and other commodity markets, a positive shock will increase volatility more than a negative one. This finding will be of great help to market participants for risk management in these new 'competitive' and deregulated electricity markets.

As said in the introductory section, in this paper we have focussed on the T-ARSV model, never used for analyzing the leverage effect in the electricity spot

markets. However, the estimation of the A-ARSV model in the electricity market context is still a pending assignment. What is more, the competition between threshold stochastic volatility models and stochastic volatility models with only correlation is a challenging future task. In other words, the challenge is to give an answer to Smith (2009) question: "Asymmetry in Stochastic Volatility Models: Threshold or Correlation?" in the context of the wholesale electricity spot markets. But, of course, the most promising and challenging avenue of research is to estimate in this markets the Smith (2009) general or augmented model with both threshold effects and correlated innovations.

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