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Path Based SDA with additional information of the dependent variable

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ABSTRACT

Structural decomposition analysis (SDA) has been widely used to assess the relative importance of effects that together constitute a change in the variable of interest. A well-known problem of SDA is that the results often depend strongly on the specific decomposition formula chosen, whereas numerous formulae are equivalent from a theoretical point of view. This non-uniqueness problem is often solved rather pragmatically, by reporting an average of (a subset of) all possible formulae. In previous works the Path Based SDA methodology has been proposed as an alternative approach to these average solutions. This technique allowed the incorporation of some additional information of the factors to choose a specific decomposition formula. This paper suggests that additional information of the variable of interest can be used with this same purpose too. We illustrate the method empirically by investigating the sources of growth in sectoral labor levels in Spain, 1986-1994.

Keywords: Structural decomposition analysis, maximum entropy, labor, Spain.

Path Based SDA con información adicional de la variable dependiente

RESUMEN

El Análisis de Descomposición Estructural (SDA) ha sido ampliamente empleado para cuantificar la importancia relativa de los diferentes efectos que conjuntamente resultan en un cambio en cierta variable de interés. Un problema reconocido de las técnicas SDA es que los resultados frecuentemente dependen en gran medida de la fórmula de descomposición elegida, existiendo numerosas alternativas equivalentes desde un punto de vista teórico. Este problema de no unicidad en las soluciones suele resolverse de un modo bastante pragmático, calculando el promedio de (un subconjunto de) todas las posibles fórmulas. En trabajos previos se ha propuesto la metodología Path Based SDA como enfoque alternativo a estas soluciones medias. Este enfoque permitía la incorporación de información adicional para elegir una forma de descomposición específica, mientras que en este artículo se sugiere que observaciones adicionales de la variable de interés puede ser empleada con este mismo propósito. Esta técnica se ilustra empíricamente analizando las fuentes del crecimiento en los niveles de empleo sectorial en España entre 1986 y 1994.

Palabras clave: Análisis de Descomposición Estructural (SDA), Máxima Entropía, Fuerza de trabajo, España.

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1. INTRODUCTION

Among all the techniques related to Input-output (IO) framework, structural decomposition analysis (SDA) is a somewhat new methodology that has gained increasing popularity recently, principally since the eighties. An extensive overview of the methodology and its relatively early applications was provided by Rose & Casler (1996). Some recent applications of different sorts include De Haan (2001), Hoekstra & Van den Bergh (2003) and Dietzenbacher *et al*. (2000, 2004). Dietzenbacher & Los (1998, 2000) showed that SDA results should be taken with care, because several methodological problems affect to the techniques employed mainly related to the non-uniqueness in its solutions, which is not only a theoretical issue: relative contributions of distinct sources of change depend considerably on the specific decomposition form chosen.

Dietzenbacher & Los (1998) showed that the number of theoretically equivalent forms amounts to $n!$, in which n represents the number of distinct sources of change. Their admittedly pragmatic solution is to present averages of results obtained for all decomposition forms or for a well-defined small subset of forms. In Fernandez's PhD thesis (2004) the Path Based method was proposed: this work argued that some available additional information could be used to divide the interaction terms in a way that fits the data better than implied by simply taking averages. The additional data were used in a Maximum Entropy (ME) estimation procedure to arrive at parameter estimates that determine the temporal paths followed by the factors. These estimates specify a unique division of the interaction terms.[1] Basically, the additional information considered in this approach were data of some of the factors involved in the decomposition problem for periods in-between the initial and final time period. In this paper we extend this approach, considering now the possibility that the only available information concerns the dependent variable whose temporal change we want to decompose.

The paper is organized as follows. In Section 2, we briefly present the "non-uniqueness" problem in SDA in formal terms by means of a simple decomposition analysis with two determinants; secondly, extending this analysis to the general case; and, finally, mentioning solutions proposed previously in the IO literature. Section 3 shows the basis of the so-called Path Based (PB) SDA, which offers a much broader class of solutions than those introduced in Section 2. One particular solution is given by a specific division of the interaction terms, and divisions are characterized by the

[1] Maximum Entropy econometrics and strongly related Cross Entropy methods have been used in an intersectoral setting before. See, for example, Golan *et al*. (1994) and Robinson *et al*. (2001) for examples where missing data in input-output tables and social accounting matrices are estimated.

parameters that appear in the temporal paths followed by the determinants. If these parameters can be estimated, we will obtain a unique solution to the decomposition, solving the non-uniqueness problem. In Section 4, the principles of ME estimation are highlighted, and we show how ME estimation techniques can be used to estimate the parameters of interest. Section 5 studies how additional information of the dependent variable can be used to implement the ME approach. In Section 6 we present an empirical illustration of the approach. We will study changes in labor requirements in Spanish sectors between 1986 and 1994. Our aim is to assess the importance of sector-specific changes in the workforce use per unit of output on the one hand, and structural effects as a consequence of changes in the matrix of input coefficients and the vector of final demands on the other. We show that the contributions obtained often deviates substantially from the average solutions analyzed by Dietzenbacher & Los (1998), among others. Moreover, we also compare the outcomes obtained by different approaches with an annual average decomposition, in order to check which of them yields the closest solution. Section 7 presents the main conclusions of the paper.

2. SDA AND THE NON-UNIQUENESS PROBLEM

From the most basic equation is **q**=**Lf** in IO analysis, where sectoral gross output levels **q** are expressed as the product of the Leontief inverse matrix **L** and the vector of sectoral final demand levels **f**, the objective of SDA is to measure the part of the variations in **q** that can be attributed to differences in **L** and the part caused by differences in **f**. This problem appears very often in other subdisciplins in economics[2], so we will explain the more traditional approaches and our new approach in terms of a more general notation.

The starting point is a dependent variable z defined as the product of a set of n factors (or, determinants)[3] $x_1, x_2, ..., x_n$. That is:

$$z = x_1 x_2 ... x_n \tag{1}$$

A fundamental assumption is that the factors can be assumed to be independent, not only in a mathematical sense (see Dietzenbacher and Los, 2000, for an account of problems related to mathematical dependency of determinants) but also from an

[2] Actually, Fernández (2004, Chapter 4) offers an application of our methodology to a shift-share analysis of employment growth in Spanish regions.

[3] The variables and the factors can be represented by scalars, vectors and/or matrices. Throughout the paper, we adopt the convention that scalars are represented by italic lowercase symbols, (column) vectors by lowercase bold symbols and matrices by bold capitals. Primes denote transposition and hats indicate diagonal matrices.

economic-theoretical viewpoint. That is, each determinant could change without a necessarily accompanying change in the values of one or more of the other determinants. Without loss of generality, we will assume that the difference in z to be studied relates to a difference over time. Denoting the value of z in the initial period 0 by z^0 and its value in the final period 1 as z^1, we can write:

$$z^0 = x_1^0 x_2^0 \dots x_n^0 \tag{2}$$

$$z^1 = x_1^1 x_2^1 \dots x_n^1 \tag{3}$$

To decompose the change in z, either additive or multiplicative approaches can be chosen. Although this last approach is gaining popularity in IO analysis since the paper by Dietzenbacher *et al*. (2000), we will focus on the probably more popular additive decomposition form, which is based on the *differences* between the left-hand sides and the right-hand sides of equations (3) and (2). We obtain:

$$\Delta z = z^1 - z^0 = x_1^1 x_2^1 \dots x_n^1 - x_1^0 x_2^0 \dots x_n^0 \tag{4}$$

The objective of additive decomposition analyses is now to express the value of the left-hand side as the sum of the respective effects of every determinant and to explain the nature of the non-uniqueness problem, we rely on the case in which n=2. For notational convenience, we will denote the factors by x and y. Hence, the temporal change in z is:

$$\Delta z = z^1 - z^0 = x^1 y^1 - x^0 y^0 \tag{5}$$

Now, by adding and subtracting $x^0 y^1$ in (5), we obtain:

$$\Delta z = x^1 y^1 - x^0 y^0 + x^0 y^1 - x^0 y^1 = \left(x^1 - x^0\right) y^1 + x^0 \left(y^1 - y^0\right) \tag{6}$$

and:

$$\Delta z = (\Delta x) y^1 + x^0 (\Delta y) \tag{7}$$

The first term on the right side of (7) represents the effect of changes in x to the actual change in z, and the second term quantifies the contribution of changes in variable y. The problem arises because different contributions could have been obtained if we had added and subtracted $x^1 y^0$ in (5) instead of $x^0 y^1$. In this case, we would have obtained:

$$\Delta z = (\Delta x) y^0 + x^1 (\Delta y) \tag{8}$$

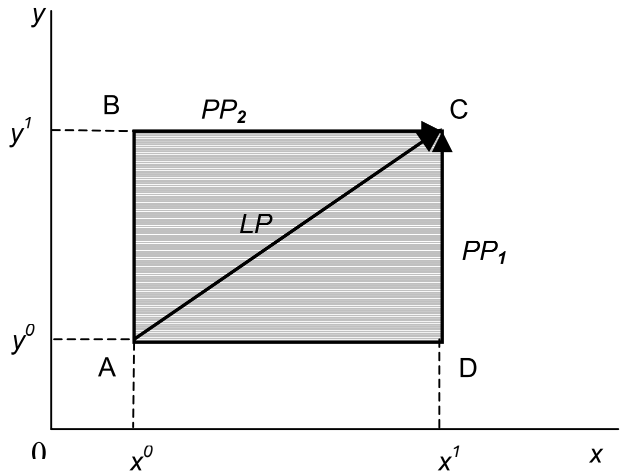
The contributions of changes in x and y as obtained by expressions (7) and (8) can differ quite a bit and choosing one of them is an arbitrary decision.[4] As a pragmatic solution, authors have traditionally applied average solutions of expressions (7) and (8). As Dietzenbacher & Los (1998) pointed out, this is equal to using midpoint weights if and only if two determinants are considered:

$$\Delta z = \Delta x y^{(½)} + x^{(½)} \Delta y \tag{9}$$

where, $x^{(½)} = \dfrac{x^0 + x^1}{2}$ and $y^{(½)} = \dfrac{y^0 + y^1}{2}$

This discussion is graphically summarized by Figure 1, which was originally proposed by Sun (1998). The whole issue is about the treatment of the upper right rectangle (ABCD), the interaction effect. Equation (7) suggests attributing it completely to the change in x, whereas equation (8) would attribute it completely to the change in y. Consequently, the contributions for x and y obtained by both expressions can imply remarkable differences, which depend on the size of the interaction term $\Delta x \Delta y$.

Figure 1. Polar and straight-line paths



[4] We only consider "exhaustive" decomposition forms, which implies that the full effect is attributed to changes in the exogenous determinants. An example of a "non-exhaustive" or "approximate" (Dietzenbacher & Los, 1998) decomposition form is $\Delta z \approx (\Delta x) y^0 + x^0 (\Delta y)$. To obtain an exact decomposition an interaction term is required, i.e., $\Delta z = (\Delta x) y^0 + x^0 (\Delta y) + (\Delta x)(\Delta y)$. The last term is often labelled the "interaction effect". In some cases, approximate forms may be preferred over exhaustive forms, for example if a clear economic interpretation can be given to the interaction term. If $n > 2$, however, approximate decompositions will contain a number of interaction terms, for which no straightforward interpretation is available. In such cases, we feel that exhaustive decomposition forms are most appropriate.

The specification of a temporal path for the determinants implies a particular decomposition form to split-up the interaction term. We will get back to the issue of temporal paths in much more detail in the next section. For now, it should be noted that path PP_1 would mean that the effect of determinant x would be $(\Delta x)y^0$, and the effect of determinant y would be $x^1(\Delta y)$. If we suppose that the temporal path between the initial and the final period is path PP_2, the respective contributions for determinants x and y would be $(\Delta x)y^1$ and $x^0(\Delta y)$. Taking the average of these two alternative paths would imply an equal division of the interaction rectangle. It can easily be seen that taking the midpoint weights would yield an identical result. This result is also attained by Sun's (1998) method, which amounts to attribute halves of the interaction effect to the effects of changes in the two determinants. This amounts to drawing a straight line (LP) from (x^0, y^0) to (x^1, y^1).[5]

In the general case, in which z is the product of n determinants, the number of possible basic decompositions such as those corresponding to PP_1 and PP_2 is increased; now being equal to the number of possible permutations for n variables. Therefore, $n!$ forms could be obtained to decompose the change Δz. Specific cases among these are:

$$\Delta z = (\Delta x_1)x_2^0 \ldots x_n^0 + x_1^1(\Delta x_2)\ldots x_n^0 + \ldots + x_1^1 x_2^1 \ldots (\Delta x_n) \tag{10}$$

$$\Delta z = (\Delta x_1)x_2^1 \ldots x_n^1 + x_1^0(\Delta x_2)\ldots x_n^1 + \ldots + x_1^0 x_2^0 \ldots (\Delta x_n) \tag{11}$$

These expressions are usually called "polar decompositions" (Dietzenbacher & Los, 1998), because the expressions for the effects are characterized by identical indexes for all determinants on both the left hand-side and right hand-side of the Δx_i factor.[6] The absence of uniqueness in the solutions leads to the arbitrary choice for one of the $n!$ possibilities, or alternatively one could obtain an average solution. As Dietzenbacher & Los (1998) showed, the average of the two polar decompositions is usually very close to the average taken over all $n!$ forms. They also show that a midpoint weighted formula is not exhaustive if $n>2$. In the next sections, we will study the main features of a general method of decomposition that overcomes many of the limitations of the SDA approaches discussed so far. It allows us to obtain non-arbitrary solutions to measure the effects of the determinants of a change.

[5] Sun's (1998) straight line can be considered as a special case of the continuous-time approach we will discuss below. Sun himself refers to his solution as the implication of a "jointly created and equally distributed" principle (Sun, 1998, p. 88).

[6] In fact, the decomposition forms corresponding to PP_1 and PP_2 also fulfill this property. We will therefore denote such paths as "polar paths" (PP).

3. THE PATH BASED APPROACH

In this section, a framework for an alternative decomposition method will be sketched. It builds on the earlier work by Vogt (1978), where the relevance of the temporal paths of the factors for the measurement of their contributions was pointed out. It is also connected with more recent works by Hoekstra & Van den Bergh (2002) and, in particular, Harrison *et al*. (2000), who introduced the basics of what we will call the Path Based (PB) approach. The alternative setup starts from the premise that both the value of z and the value of the determinants x_i have changed continuously over time, between time 0 and time 1. Hence, we can write:

$$z(t) = x_1(t)x_2(t)...x_n(t) \tag{12}$$

and, assuming differentiability of each $x_i(t)$ an infinitesimal change in z can be expressed as

$$dz = \frac{\partial z}{\partial x_1}\frac{dx_1}{dt}dt + ... + \frac{\partial z}{\partial x_n}\frac{dx_n}{dt}dt \tag{13}$$

Finally, the total change in z can be expressed as the sum of all the infinitesimal changes between time 0 and time 1:

$$\Delta z = \int_{t=0}^{t=1} \frac{dz}{dt}dt = \int_{t=0}^{t=1} \sum_{i=1}^{n} \frac{\partial z}{\partial x_i}\frac{dx_i}{dt}dt \tag{14}$$

The effects of the determinants x_i can now be written as:

$$\Delta x_i \text{ Effect} = \int_{t=0}^{t=1} \frac{\partial z}{\partial x_i}\frac{dx_i}{dt}dt = \int_{t=0}^{t=1} \prod_{j \neq i}^{n} x_j \frac{dx_i}{dt}dt \tag{15}$$

Equation (15) shows that the derivatives of the determinants x_i to time t play an important role in the size of the effects attributed to changes in these determinants. Consequently, the specification of the temporal path that each factor follows between initial and final periods, $x_i(t)=f_i(t)$, can have a big impact on the measurement of their effects that together add up to the variation in z. Harrison *et al*. (2000) proposed the solution arrived at by assuming straight-line paths of the variables x_i:

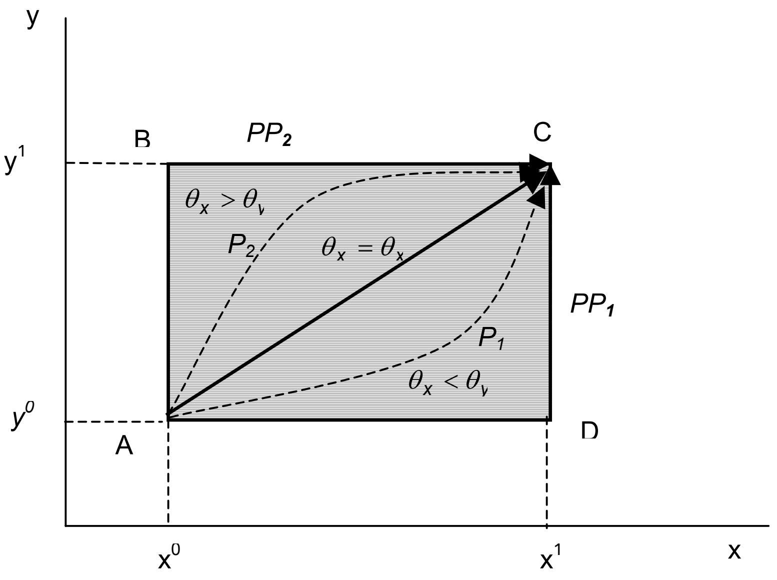
$$x_i(t) = x_i^0 + \left(x_i^1 - x_i^0 t\right) = x_i^0 + (\Delta x_i)t \tag{16}$$

Actually, this approach yields the same solution as Sun's (1998) 'equal shares' method. However, empirical values of the variables x and y at, for example, t=0.5 might be such that the straight line assumption is very unlikely to be tenable. In previous works, we suggested a method to take such information explicitly into account in attributing parts of the interaction effects to the effects of the respective determinants. The methodological innovation proposed was to relax the strict assumption of a straight line, by considering more flexible forms for the functions $f_i(t)$. In order to preserve possibilities to estimate the parameters that characterize the time-paths of the variables, we choose to consider a specific class of monotonic functions:

$$x_i(t) = x_i^0 + (\Delta x_i)t^{\theta_i}; \ \forall \theta_i > 0 \tag{17}$$

The class of paths considered contains all possible monotonic paths for x_0 to x_1 that do not have inflexion points. This is a limitation for sure, because our class of paths does not cover those that contain values that are below the initial value or exceed the final value (assuming, without loss of generalization that x_1 is larger than x_0). Obviously, the temporal path of x_i will be a straight line if θ_i equals 1. If this holds for all i (i=1, ..., n), the solution obtained by the method introduced here will be identical to Harrison's *et al*. (2000) solution. By plotting a diagram for two determinants comparable to Figure 1, we can show that the class of time paths implied by the still relatively simple expression in equation (17) comprises a nicely defined set of time paths (see Figure 2).

Figure 2. Generalized monotonic temporal paths



The basic idea is that the specific path implied by the parameter values θ_i determine the shares of the interaction effect that is attributed to the distinct determinants. The "polar" paths PP_1 ($\theta_x/\theta_y \to 0$) and PP_2 ($\theta_x/\theta_y \to \infty$), and the straight-line path ($\theta_x/\theta_y = 1$) are included as special cases of this general class. P_1 and P_2 are intermediate cases. In a situation like the one represented by P_1, a larger part of the interaction effect is attributed to determinant y than if a situation better reflected by P_2 would occur.

For the most general case in which a change in z is decomposed into the effects of n determinants x_i (see equation (12)), the expression for the respective contributions for any possible set of n time paths was already given in equation (15). Substituting the more specific temporal paths assumed in equation (17) into equation (15), we can write

$$\Delta x_i \text{ Effect} = \int_{t=0}^{t=1} \prod_{j \neq i}^{n} x_j \frac{dx_i}{dt} dt = \left[\prod_{j<i}^{i-1} x_j^0\right](\Delta x_i)\left[\prod_{j>i}^{n} x_j^0\right] + \tag{18a}$$

$$+ \sum_{j \neq i}^{n} \left[\frac{\theta_i}{\theta_i + \theta_j} \prod_{k<i}^{i-1} x_k^0 (\Delta x_i) \prod_{i<k<j}^{j-1} x_k^0 (\Delta x_j) \prod_{k>j}^{n} x_k^0 \right] + \tag{18b}$$

$$+ \sum_{j \neq i}^{n} \sum_{l \neq j,i}^{n} \left[\frac{\theta_i}{\theta_i + \theta_j + \theta_l} \prod_{k<i}^{i-1} x_k^0 (\Delta x_i) \prod_{i<k<j}^{j-1} x_k^0 (\Delta x_j) \prod_{j<k<l}^{l-1} x_k^0 (\Delta x_l) \prod_{k>l}^{n} x_k^0 \right] + \tag{18c}$$

$$+ \frac{\theta_i}{\sum_{j=1}^{n} \theta_j} \left[\prod_{j=1}^{n} (\Delta x_j)\right] \tag{18d}$$

The first term in this sum shows the smallest contribution for determinant x_i, which is given by its growth Δx_i weighted by the initial values of the other variables.[7] It does not contain any part of the interaction effects. The remaining terms show a set of interaction effects between the growths of groups of determinants, also weighted by the initial values of the remaining determinants. The distribution of these joint effects clearly depends on the $\grave{e}_i$ values. Multiple joint effects between the determinants exist. More specifically, there are $\binom{n-1}{1}$ possibilities of interaction between x_i and each one of the remaining $n-1$ determinants, $\binom{n-1}{2}$ terms measuring the joint effect of

[7] For the sake of simplicity, let us hereafter suppose a situation in which $\Delta x_i \geq 0$; $i = 1,\ldots,n$.

x_i with groups of $n-2$ determinants, etc. In general, in the expression for the effect of x_i there will be $\binom{n-1}{k}$ terms for the joint effects with groups of *k* determinants. The last terms (in equation 18d), shows the part of the joint contribution of all the determinants to the interaction effect attributed to x_i.

The importance of the values of the θ_i parameters for the measurement of the determinant's contributions is clear from equation (18). The higher the value of θ_i in comparison to the remaining parameters θ_j, the greater the portions of the interaction effects attributed to x_i and, thus, the greater its contribution to the whole change in variable *z*. To illustrate this idea, it is helpful to give extreme values to a parameter θ_i. Let us suppose firstly that θ_i tends to its minimum value, *i.e.* we are supposing that it is very close to zero. In this case we obtain:

$$\lim_{\theta_i \to 0} \Delta x_i \text{ Effect} = \left[\prod_{j<i}^{i-1} x_j^0\right](\Delta x_i)\left[\prod_{j>i}^{n} x_j^0\right] = x_1^0 x_2^0 \ldots x_{i-1}^0 (\Delta x_i) x_{i+1}^0 \ldots x_n^0 \tag{19}$$

This would be the case when the effect of changes in variable x_i is at its smallest, because we are weighting it by the remaining determinants at their initial values. It should be noted that equation (19) is one of the $n!$ feasible solutions obtained by SDA. The opposite situation will happen if we suppose that parameter θ_i has a much higher value than the rest of parameters θ_j. Then, the contribution of x_i will be:

$$\lim_{\theta_i \to \infty} \Delta x_i \text{ Effect} = x_1^1 x_2^1 \ldots x_{i-1}^1 (\Delta x_i) x_{i+1}^1 \ldots x_n^1 \tag{20}$$

In such a case, the contribution of x_i to changes in variable *z* is as large as it can be, since we are weighting its variation by the remaining determinants measured at their final values. Between these two extreme situations there exists an infinite range of possible contributions for each determinant, which depend on the value of parameters θ_i. All solutions obtained by SDA techniques are included in this range.

As we mentioned before, it is nowadays a common practice in SDA analyses to present averages over decomposition forms. The average over all *n!* decomposition forms could be obtained by the PB approach as well. If we would not have any information on the evolution of the determinants over time other than the initial and the final observation, it would be most plausible to assume that the temporal path parameters are equal to each other ($\theta_1 = \theta_2 = \ldots = \theta_n$). According to equation (18) we would find

$$\Delta x_i \text{ Effect} = \int_{t=0}^{t=1} \prod_{j \neq i}^{n} x_j \frac{dx_i}{dt} dt = \left[\prod_{j<i}^{i-1} x_j^0\right](\Delta x_i)\left[\prod_{j>i}^{n} x_j^0\right] + \tag{21a}$$

$$+ \sum_{j \neq i}^{n} \left[\frac{1}{2} \prod_{k<i}^{i-1} x_k^0 (\Delta x_i) \prod_{i<k<j}^{j-1} x_k^0 (\Delta x_j) \prod_{k>j}^{n} x_k^0\right] + \tag{21b}$$

$$+\sum_{j\neq i}^{n}\sum_{l\neq j,i}^{n}\left[\frac{1}{3}\prod_{k<i}^{i-1}x_k^0(\Delta x_i)\prod_{i<k<j}^{j-1}x_k^0(\Delta x_j)\prod_{j<k<l}^{l-1}x_k^0(\Delta x_l)\prod_{k>l}^{n}x_k^0\right]+ \tag{21c}$$

$$+\frac{1}{n}\left[\prod_{j=1}^{n}(\Delta x_j)\right] \tag{21d}$$

The interaction effects are thus shared proportionally to the changes in the values of the determinants. This is identical to the solution proposed by Sun (1998) discussed in the previous section. In spite of the similarity between the numerical outcomes for the mean of the two polar decompositions only and the mean of all *n!* decompositions (Dietzenbacher & Los, 1998), the mean of the polar decompositions cannot be obtained by means of specifying values for θ_i in the above-mentioned PB approach.[8] In the next sections we will turn to methods to infer on plausible values for θ_i, which allow us to apply equation (18) to interesting empirical problems.

4. GENERALIZED MAXIMUM ENTROPY ECONOMETRICS WITH NON-LINEAR CONSTRAINTS

In the previous section, we found that taking the mean contributions of all decomposition forms is the most reasonable solution to the non-uniqueness problem if the researcher has no information at all about the time paths of the determinants. In many cases, however, more information than the values of the determinants at t=0 and t=1 is available. Anyway, estimation of the parameters θ_i is generally not possible by means of classical econometric estimation procedures like least squares estimation, since the amount of data is quite limited. In Fernández *et al.* (2005) such a situation is studied when the available information is the values of one or more of the determinants at intermediate points in time. The estimation procedure followed is based on maximum entropy (ME) econometrics, a collection of tools that can be very convenient to use scarce additional information in producing estimates for the temporal path parameters θ_i.[9].

The starting point will be a random variable x which can get values $\{x_1,...,x_K\}$ with a distribution of probabilities $\mathbf{p} = p_1, p_2,..., p_K$ that is unknown for the researcher and has to be recovered. Following the formulation of Shannon (1948), the entropy of this distribution **p** will be

[8] See Fernández (2004, pp. 36-39) for a proof.

[9] See Kapur & Kesavan (1992) or Golan *et al.* (1996) for a detailed analysis of properties of the estimators obtained by means of these techniques.

$$H(\mathbf{p}) = -\sum_{i=1}^{K} p_i \, h \, p_i \tag{22}$$

The entropy measure H indicates the 'uncertainty' and reaches its maximum when **p** is a uniform distribution ($p_i = \frac{1}{K}, \forall i = 1,...,K$). If some information (*i.e.*, observations) is available, H can also be used to estimate **p**. Suppose that there are T observations $\{y_1, y_2, ..., y_T\}$ available such that

$$\sum_{i=1}^{K} p_i f_t(x_i) = y_t, \; 1 \leq t \leq T \tag{23}$$

with $\{f_1(x)\ f_2(x),...,f_T(x)\}$ a set of known functions representing the relationships between the random variable x and the observed data $\{y_1, y_2, ..., y_T\}$. In such a case, the ME principle can be applied to recover the unknown probabilities. This principle is based on the selection of the probability distribution that maximizes equation (22) among all the possible probability distributions that fulfill (23). The following constrained maximization problem is posed:

$$\underset{\mathbf{p}}{Max}\ H(\mathbf{p}) = -\sum_{i=1}^{K} p_i \, h \, p_i \tag{24}$$

subject to:

$$\sum_{i=1}^{K} p_i f_t(x_i) = y_t, \; \forall t = 1,...,T$$

$$\sum_{i=1}^{K} p_i = 1$$

In this problem, the last restriction is just a normalization constraint that guarantees that the estimated probabilities sum to one, while the first T restrictions guarantee that the recovered distribution of probabilities is compatible with the data for all T observations. By solving this program, one "chooses" the estimates $\hat{\mathbf{p}}$ among all the distribution probabilities consistent with the information available, being $\hat{\mathbf{p}}$ the one that maximizes the entropy. In other words, $\hat{\mathbf{p}}$ is the one that implies using only the amount of information involved in the T observations. Hence, in situations in which the number of observations is not large enough to apply econometrics based on limit theorems, this approach can be used to obtain robust estimates of unknown parameters.[10]

[10] Golan *et al.* (1996, p. 12) contains a simple, classic example of this technique, the so-called "dice problem". A disadvantage of ME estimators is that comparisons of means and variances of estimators are not possible. Such comparisons are common practice in classical least squares and maximum likelihood econometrics.

The above-sketched procedure has been generalized (GME) to the estimation of unknown parameters for linear models (see Paris & Howitt, 1998; or Golan *et al.*, 2001 as examples), although some recent works have advocated the use of GME when the model to estimate is non-linear (see Grendar & Grendar, 2004). To illustrate this in a simple way, let us suppose that the model to estimate has the following form:

$$\mathbf{y} = \mathbf{x}_1^{(\theta_1)} + \mathbf{x}_2^{(\theta_2)} + \ldots + \mathbf{x}_n^{(\theta_n)} + \mathbf{e} \tag{25}$$

where $\mathbf{y}$ is a $(T \times 1)$ vector of observations for y, $\mathbf{x_i}$ is a $(T \times 1)$ vector of observations for the x_i variables ($\forall i = 1,\ldots,n$), θ_i is an unknown parameter to be estimated ($\forall i = 1,\ldots,n$), and $\mathbf{e}$ is a $(T \times 1)$ vector reflecting the random term of the linear model. For each θ_i, it will be assumed that there is some information about its $M \geq 2$ possible realizations by means of a 'support' vector $\mathbf{b}' = (b_1,\ldots,b^*,\ldots,b_M)$, the elements of which are symmetrically distanced around a central value $\theta_i = b^*$ (the prior expected value of the parameter), with corresponding probabilities $\mathbf{p_i'} = (p_{i1},\ldots,p_M)$. For the sake of convenient exposition, it will be assumed that the M values are the same for every parameter, although this assumption can easily be relaxed. Consequently, Now, each parameter θ_i can be written as

$$\theta_i = \sum_{m=1}^{M} p_{im} b_m, \quad \forall i = 1,\ldots,n \tag{26}$$

For the random terms, a similar approach is chosen. To express the lack of information about the actual values contained in $\mathbf{e}$, we assume a distribution for each e_t, with a set of $H \geq 2$ values $\mathbf{v}' = (v_1,\ldots,v_H)$ with respective probabilities $\mathbf{w_t'} = (w_{t1}, w_{t2},\ldots,w_{tH})$.[11] Hence, we can write

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ \ldots \\ e_T \end{bmatrix} = \mathbf{Vw} = \begin{bmatrix} \mathbf{v}' & \mathbf{0} & . & \mathbf{0} \\ \mathbf{0} & \mathbf{v}' & . & \mathbf{0} \\ . & . & . & . \\ \mathbf{0} & \mathbf{0} & . & \mathbf{v}' \end{bmatrix} \begin{bmatrix} \mathbf{w_1} \\ \mathbf{w_2} \\ \ldots \\ \mathbf{w_T} \end{bmatrix} \tag{27}$$

and the value of the random term for an observation t equals

$$e_t = \mathbf{v}'\mathbf{w_t} = \sum_{h=1}^{H} v_h w_{th}; \quad \forall t = 1,\ldots,T \tag{28}$$

And, consequently, equation (25) can be transformed into

[11] Usually, the distribution for the errors is assumed symmetric and centered about 0, therefore $v_1 = -v_J$

$$\mathbf{y} = \mathbf{x}_1^{\left(\sum_{m=1}^{M} b_m p_{1m}\right)} + \mathbf{x}_2^{\left(\sum_{m=1}^{M} b_m p_{2m}\right)} + \ldots + \mathbf{x}_n^{\left(\sum_{m=1}^{M} b_m p_m\right)} + \mathbf{vW} \tag{29}$$

and the ME program to estimate the $n+T$ probability distributions is the following:

$$\underset{\mathbf{p},\mathbf{w}}{Max}\, H(\mathbf{p},\mathbf{w}) = -\sum_{i=1}^{n}\sum_{m=1}^{M} p_{im}\, ln(p_{im}) - \sum_{t=1}^{T}\sum_{h=1}^{H} w_{th}\, ln(w_{th}) \tag{30}$$

subject to:

$$\sum_{m=1}^{M} p_{im} = 1, \ \forall i = 1,\ldots,n$$

$$\sum_{h=1}^{H} w_{th} = 1, \ \forall t = 1,\ldots,T$$

$$y_t = \sum_{i=1}^{n}\left[x_t^{\left(\sum_{m=1}^{M} b_m p_{im}\right)} \right] + \sum_{h=1}^{H} v_h w_{th} =, \ \forall t = 1,\ldots,T$$

The estimated probabilities allow us to obtain estimations for the unknown parameters. The estimated value of θ_i will be[12],[13]:

$$\hat{\theta}_i = \sum_{m=1}^{M} \hat{p}_{im} b_m \quad , \ \forall i = 1,\ldots,n \tag{31}$$

This approach can be applied to the decomposition problem studied in the previous section, since limited additional information would enable us to obtain estimates of the parameters that determine the contribution of each determinant to the total change that has actually been observed. In the next section a situation with availability of additional data for *z(t)* will be considered, as well as the way to estimate the effects of the factors to the total change Δz using this technique.

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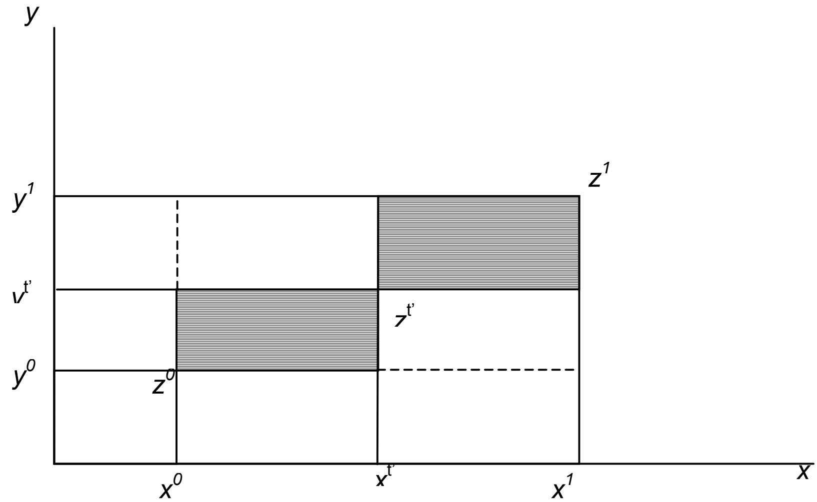
[12] The construction of the vector b is based on the researcher's prior knowledge (or beliefs) about the parameter. Golan *et al*. (1996, chapter 8) devote more attention to consequences of choices concerning the elements of the vector b. An almost universal result is that wider bounds can be used without substantial consequences for the characteristics of the estimators.

[13] Fernández (2004, pp. 69) proves that the solution of the constrained maximization problem (36) without additional information yields estimates equal to the expected value b^* of the prior distribution.

5. INCORPORATING ADDITIONAL INFORMATION OF *Z(T)*

If there is some available additional information of intermediate periods between $t=0$ and $t=1$, this can help to reduce the non-uniqueness problem in SDA. Suppose that for the simplest case where $z(t)=x(t)y(t)$, we know the values of $x(t')$ and $y(t')$, being $0<t'<1$. In such a case, see that if we can incorporate the additional information and consider a two-stage decomposition:

Figure 3. Two-stage decomposition



The sum of the interaction terms for the two stages of the decomposition (two grey shaded areas) is smaller than the interaction term $\Delta x\Delta y$ that appears in Figures 1 y 2. If the number of stages is increased, *i. e.* if the intermediate periods with observations for factors x and y increases, the size of the interaction terms decreases and this also reduces the seriousness of the non-uniqueness problem[14]. Unfortunately, such a solution is not always possible because the lack of regular information of IO tables for many economies makes this "dynamic" approach often unfeasible. In previous works (Fernández, 2004; Fernandez *et al.*, 2005), the use of additional information of some of the factors has been studied. This paper suggests a different perspective, considering the possibility of including intermediate observations not of the factors, but concerning the dependent or endogenous variable z.

To illustrate this idea in a simple way, let us continue with the simplest case with two factors x and y. Suppose that for an intermediate period t' we have collected the

[14] Note that in the hypothetical case when with an infinite number of intermediate points, the interaction terms would vanish and there would be a unique decomposition form.

value *z(t´)*, although *x(t´)* and *y(t´)* are both unknown. Obviously, this situation disables a two-stage decomposition like the represented in Figure 3, but the additional information *z(t´)* can be used somehow to obtain a decomposition different from the average of equations (7) and (8), which would be the most appropriate solution if no additional information is available. Applying the PB approach to this context, we will have the following temporal paths for factors *x* and *y*:

$$x(t) = x^0 + (\Delta x)t^{\theta_x};\ \forall \theta_x > 0 \tag{32}$$

$$y(t) = y^0 + (\Delta y)t^{\theta_y};\ \forall \theta_y > 0 \tag{33}$$

And, consequently:

$$z(t) = x(t)y(t) = x^0y^0 + x^0(\Delta y)t^{\theta_y} + (\Delta x)y^0t^{\theta_x} + (\Delta x)(\Delta y)t^{(\theta_x+\theta_y)};\ \forall \theta_x,\theta_y > 0 \tag{34}$$

If we include a stochastic component ε_t that allows *z* to diverge from the deterministic path (34), we obtain[15]

$$z(t) = x^0y^0 + x^0(\Delta y)t^{\theta_y} + (\Delta x)y^0t^{\theta_x} + (\Delta x)(\Delta y)t^{(\theta_x+\theta_y)} + \varepsilon_t;\ \forall \theta_x,\theta_y > 0 \tag{35}$$

Or, equivalently

$$\Delta z(t) = z(t) - z^0 = x^0(\Delta y)t^{\theta_y} + (\Delta x)y^0t^{\theta_x} + (\Delta x)(\Delta y)t^{(\theta_x+\theta_y)} + \varepsilon_t;\ \forall \theta_x,\theta_y > 0 \tag{36}$$

Equation (36) is a non-linear model with two parameters to be estimated. Defining $\theta_x = \mathbf{b'p}_x = \sum_{m=1}^{M} b_m p_{xm}$ and $\theta_y = \mathbf{b'p}_y = \sum_{m=1}^{M} b_m p_{ym}$ where $\mathbf{p_x}$ and $\mathbf{p_y}$ are unknown probability distributions, equation (36) can be written as[16]

$$\Delta z(t) = x^0(\Delta y)t^{\left(\sum_{m=1}^{M} b_m p_{ym}\right)} + (\Delta x)y^0t^{\left(\sum_{m=1}^{M} b_m p_{xm}\right)} + (\Delta x)(\Delta y)t^{\left(\sum_{m=1}^{M} b_m p_{xm} + \sum_{m=1}^{M} b_m p_{ym}\right)} + \sum_{j=1}^{J} v_j w_t \tag{37}$$

Hence, it is possible to apply the Maximum Entropy estimation technique for non-linear relationships analyzed in the previous section. Upon having estimated both parameters, it is immediate to obtain the estimated respective contributions of changes in the determinants by substituting their values in the following equations

$$\Delta x \text{ Effect} = (\Delta x)y^0 + \frac{\theta_x}{\theta_x + \theta_y}(\Delta x)(\Delta y) \tag{39}$$

[15] We assume that $\varepsilon_t = 0$ in the final period. This ensures that $z(t)$ has value z^1 in this period.

[16] Note that in (41) the stochastic term ε_t is also defined in the same way as equation (29).

$$\Delta y \text{ Effect} = x^0(\Delta y) + \frac{\theta_y}{\theta_x + \theta_y}(\Delta x)(\Delta y) \tag{40}$$

which are the reduced versions for the two-factors case of equation (18).

6. ILLUSTRATION: DECOMPOSITION OF SECTORAL WORKFORCE REQUIREMENTS IN THE SPANISH ECONOMY

We apply the technique developed in the previous sections to study the contributions of three determinants to changes in real sectoral labor requirements in Spain, over the period 1986-1994. It should be emphasized that the aim of this section is not so much to provide a deep analysis of the dynamics of Spanish labor requirements, but rather to provide an illustration of the methods proposed in this paper. Specifically, in this section we will compare by means of an empirical example the outcomes obtained by the PB methodology proposed with some other more traditional decomposition approaches. The required data were taken from 21-sector input-output tables for these years, expressed in constant prices of 1986. The intermediate blocks of the tables contain domestic deliveries only. Appendix A contains detailed information about how we treated the basic data to arrive at the data used in the analysis outlined below. The starting point is an input-output model that expresses the vector ($k \times 1$) of sectoral labor requirements **z** as the product of three factors, i.e. labor requirements per unit of gross output **u** (included as a diagonal $k \times k$ matrix), the Leontief inverse matrix **L** and the vector of final demands **f**.

$$\mathbf{z} = \hat{\mathbf{u}}\mathbf{L}\mathbf{f} \tag{40}$$

and the objective is to decompose the total change $\Delta\mathbf{z}$ into the following three components:

$$\mathbf{z}^{94} - \mathbf{z}^{86} = \Delta\mathbf{z} = \Delta\mathbf{u} \text{ Effect} + \Delta\mathbf{L} \text{ Effect} + \Delta\mathbf{f} \text{ Effect} \tag{41}$$

We assume the following temporal paths for the elements of the factors:

$$u_k(t) = u_k^{86} + (\Delta u_k)t^{\theta_{u_k}} \; ; k=1, ..., 21 \tag{42a}$$

$$l_{kj}(t) = l_{kj}^{86} + (\Delta l_{kj})t^{\theta_{l_{kj}}} \; ; k=1, ..., 21; j=1, ..., 21 \tag{42b}$$

$$f_k(t) = f_k^{86} + (\Delta f_k)t^{\theta_{f_k}} \; ; k=1, ..., 21 \tag{42c}$$

According to equation (21), the contributions of changes in elements of **u**, **L** and **f** to the changes in **c** can be written as[17]

$$\Delta\hat{\mathbf{u}}\ \text{Effect} = (\Delta\hat{\mathbf{u}})\mathbf{L}^{86}\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+L}} \circ (\Delta\hat{\mathbf{u}})(\Delta\mathbf{L})\right]\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+f}} \circ (\Delta\hat{\mathbf{u}})\mathbf{L}^{86}\right](\Delta\mathbf{f}) + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+L+f}} \circ (\Delta\hat{\mathbf{u}})(\Delta\mathbf{L})\right](\Delta\mathbf{f}) \tag{43a}$$

$$\Delta\mathbf{L}\ \text{Effect} = \hat{\mathbf{u}}^{86}(\Delta\mathbf{L})\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{u+L}} \circ (\Delta\hat{\mathbf{u}})(\Delta\mathbf{L})\right]\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{L+f}} \circ \hat{\mathbf{u}}^{86}(\Delta\mathbf{L})\right](\Delta\mathbf{f}) + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{u+L+f}} \circ (\Delta\hat{\mathbf{u}})(\Delta\mathbf{L})\right](\Delta\mathbf{f}) \tag{43b}$$

$$\Delta\mathbf{f}\ \text{Effect} = \hat{\mathbf{u}}^{86}\mathbf{L}^{86}(\Delta\mathbf{f}) + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{u+f}} \circ (\Delta\hat{\mathbf{u}})\mathbf{L}^{86}\right](\Delta\mathbf{f}) + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{L+f}} \circ \hat{\mathbf{u}}^{86}(\Delta\mathbf{L})\right](\Delta\mathbf{f}) + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{u+L+f}} \circ (\Delta\hat{\mathbf{u}})(\Delta\mathbf{L})\right](\Delta\mathbf{f}) \tag{43c}$$

with the matrices **Θ** defined as

$$\boldsymbol{\Theta}^{u}_{u+L} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_1}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{1K}}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{K1}}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{u}_{u+f} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{f_1}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{f_1}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{u}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_1}+\theta_{f_1}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_K}+\theta_{f_K}} \end{bmatrix}$$

$$\boldsymbol{\Theta}^{L}_{u+L} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{u_1}+\theta_{l_1}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{u_1}+\theta_{l_{1K}}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{u_1}+\theta_{l_{K1}}} & \cdots & \frac{\theta_{l_K}}{\theta_{u_K}+\theta_{l_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{L}_{L+f} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{l_1}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_K}}{\theta_{l_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{L}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{u_1}+\theta_{l_1}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_K}}{\theta_{u_K}+\theta_{l_K}+\theta_{f_K}} \end{bmatrix}$$

$$\boldsymbol{\Theta}^{f}_{u+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{u_1}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_1}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{u_K}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{f}_{L+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{l_1}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{l_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{f}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{u_1}+\theta_{l_1}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_K}+\theta_{l_K}+\theta_{f_K}} \end{bmatrix}$$

As argued before, assuming that parameters θ are the same for all the factors (i.e., $\theta_{u_k} = \theta_{l_k} = \theta_{f_k}$) for all *j* and *k* would yield Sun's (1998) solution. This would be a natural thing to do if no information would be available for the years in-between 1986 and 1994. To illustrate the techniques outlined in the previous sections, we will estimate some of the θ parameters by employing additional information of the sectoral labor requirements **z** in two intermediate years: 1989 and 1992. First, we had to decide on the values to be assigned to the a priori distributions contained in the support vector **b** and the possible realizations for the random term in vectors

[17] The symbol $\circ$ indicates element-by-element (Hadamard) multiplication.

v. The following vectors were used for all parameters throughout the empirical analyses below:[18]

b=[-5.0, -3.0, -1.0, 1.0, 3.0, 5.0, 7.0]' and **v**=[-100, -50, 0, 50, 100]'

Note that to the central value of **b**, namely b^*, is assigned a value 1 since the solution of the constrained maximization problem (36) without additional information yields estimates $\hat{\theta}$ equal to this value b^*. This means that, with lack of information for a temporal path, there are not reasons to assume a convex (i. e., $\hat{\theta} < 1$) or concave (i. e., $\hat{\theta} > 1$) path either. Let us define $\theta_{uk} = \mathbf{b'p}_{uk} = \sum_{m=1}^{M} b_m p_{ukm}$, $\theta_{ljk} = \mathbf{b'p}_{ljk} = \sum_{m=1}^{M} b_m p_{ljk\,m}$ and $\theta_{fk} = \mathbf{b'p}_{fk} = \sum_{m=1}^{M} b_m p_{fk\,m}$ where $\mathbf{p}_{uk}$, $\mathbf{p}_{lkj}$ and $\mathbf{p}_{fj}$ are unknown probability distributions. In an equivalent way as that depicted previously for the simplest two-factors case, the sectoral labor use in a sector i will be:

$$
\begin{aligned}
z_i(t) = {} & u_i^{86}\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} + u_i^{86}\sum_{k=1}^{21}(\Delta l_{ik}) f_k^{86} t^{\theta_{ljk}} + (\Delta u_i)\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} t^{\theta_k} + u_i^{86}\sum_{j=1}^{21} l_{ik}^{86}(\Delta f_k) t^{\theta_{fk}} + \\
& (\Delta u_i)\sum_{k=1}^{21}(\Delta l_{ik}) f_j^{86} t^{(\theta_{iu}+\theta_{lik})} + (\Delta u_i)\sum_{k=1}^{21} l_k^{86}(\Delta f_k) t^{(\theta_{iu}+\theta_{fk})} + u_i^{86}\sum_{k=1}^{21}(\Delta l_{ik})(\Delta f_k) t^{(\theta_{lik}+\theta_{fk})} + \\
& (\Delta u_i)\sum_{k=1}^{21}(\Delta l_{ik})(\Delta f_k) t^{(\theta_{iu}+\theta_{lik}+\theta_{fk})} + \sum_{h=1}^{H} v_h w_{th}
\end{aligned}
\tag{44}
$$

In (44) a stochastic element (last term) has been included as we commented before. In an equivalent way as that depicted in program (38) for the simplest two-factors case, the following GME program has to be solved, in which this expression (44) ends up as a constraint

$$
\underset{\mathbf{p}_u,\mathbf{p}_L,\mathbf{p}_f,\mathbf{w}}{Max}\; H(\mathbf{p}_u,\mathbf{p}_L,\mathbf{p}_f,\mathbf{w}) = -\sum_{k=1}^{21}\sum_{j=1}^{21}\sum_{m=1}^{M}\left[p_{ukm}\, ln(p_{ukm}) + p_{ljkm}\, ln(p_{ljkm}) + p_{fkm}\, ln(p_{fkm})\right] - \sum_{t=1}^{T}\sum_{h=1}^{H} w_{th}\, ln(w_{th})
\tag{45}
$$

subject to:

$$
\sum_{m=1}^{M} p_{ukm} = \sum_{m=1}^{M} p_{ljk\,m} = \sum_{m=1}^{M} p_{fk\,m} = 1, \;\forall k, j = 1,\ldots,21
$$

[18] In the empirical application we have been forced to make a couple of exceptions: due to the extremely great labor requirements observed for sectors 14 and 21 (in relative terms to the rest of the sectors), it was necessary to consider wider bound for the vector v in order to get feasible estimates. In these two cases, the bounds were enlarged and go from –150 to 150. Golan *et al*. (1996, p. 138) asserted that the estimation results are generally not very sensitive to the choice of a particular set.

$$\sum_{h=1}^{H} w_{th} = 1, \; \forall t = 1,...,T$$

$$z_i(t) = u_i^{86}\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} + u_i^{86}\sum_{k=1}^{21}(\Delta l_{ik}) f_k^{86} t^{\theta_{l_{jk}}} + (\Delta u_i)\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} t^{\theta_k} + u_i^{86}\sum_{j=1}^{21} l_{ik}^{86}(\Delta f_k) t^{\theta_{fk}} +$$
$$(\Delta u_i)\sum_{k=1}^{21}(\Delta l_{ik}) f_j^{86} t^{(\theta_{iu}+\theta_{lik})} + (\Delta u_i)\sum_{k=1}^{21} l_{ik}^{86}(\Delta f_k) t^{(\theta_{iu}+\theta_{fk})} + u_i^{86}\sum_{k=1}^{21}(\Delta l_{ik})(\Delta f_k) t^{(\theta_{lik}+\theta_{fk})} +$$
$$(\Delta u_i)\sum_{k=1}^{21}(\Delta l_{ik})(\Delta f_k) t^{(\theta_{iu}+\theta_{lik}+\theta_{fk})} + \sum_{h=1}^{H} v_h w_{th}, \; \forall i = 1,...,21; \; \forall t = 1,...,T$$

The estimates obtained here will be used to obtain the respective contributions for the effect of changes in the three factors considered.

6.1 Contributions of changes in the determinants

Table 1 reports the values of the variation in the sectoral workforce use between 1986 and 992 and the intermediate observations of this variable in 1989 and 1992; being these data obtained form the Spanish National Accounts (INE, 1990 and 1993). These observations lead to the estimates of the parameters that characterize the temporal paths (42a-42c) and they are shown in Appendix B. Table 1 also reports the results of the decomposition for changes in the three determinants, as well as ratios that compare the results with those obtained by average decompositions:

Table 1: Variation in z, values of z(t) in 1989 and 1992 and decomposition results [a]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Sector	Δz^b	*z(1989)*	*z(1992)*	*Δu Eff*	*ΔL Eff*	*Δf Eff*	ρ_u^F	ρ_L^F	ρ_f^F	ρ_u^P	ρ_L^P	ρ_f^P
s1	-543	1488	1212	-652.31	-255.23	364.54	107.21	105.58	118.67	107.75	103.00	117.51
s2	-34	150	133	-42.81	-23.04	31.85	102.78	105.12	107.71	103.42	102.70	106.77
s3	-18	84	85	-16.79	-45.06	43.85	99.74	104.71	104.74	103.31	101.96	103.31
s4	2	183	188	-36.27	-26.50	64.77	108.33	112.26	109.62	109.05	110.22	109.22
s5	-1	144	139	-54.22	-17.68	70.90	101.13	106.17	102.36	101.86	101.48	101.79
s6	25	308	296	-47.56	-19.59	92.16	105.58	107.91	104.48	106.00	105.83	104.27
s7	18	115	120	0.67	-1.65	18.99	101.16	102.40	100.16	101.19	102.43	100.17
s8	5	207	207	-80.57	-11.26	96.84	103.45	109.50	103.94	103.82	103.97	103.63
s9	-20	269	265	-173.21	-13.07	166.28	103.88	109.86	104.82	104.20	101.14	104.48
s10	22	425	425	-39.06	-13.60	74.67	103.67	105.52	102.89	103.77	104.93	102.84
s11	-80	432	393	-274.73	10.89	183.85	101.79	136.69	101.05	101.78	135.82	101.06
s12	25	167	183	-9.76	-5.54	40.30	105.41	106.97	102.19	105.52	106.58	102.16
s13	23	387	390	43.88	-62.81	41.93	104.39	106.35	104.76	105.19	107.52	105.61
s14	238	1139	1206	-5.55	-14.83	258.38	112.72	107.01	100.62	112.77	106.98	100.62
s15	336	1998	2119	-133.53	61.93	407.60	106.12	102.97	101.47	105.96	102.30	101.52
s16	188	816	831	58.82	4.78	124.40	107.43	102.05	96.76	107.39	102.82	96.75
s17	10	565	574	-175.81	25.02	160.79	100.83	103.11	100.43	100.58	99.62	100.69
s18	31	135	151	-40.52	28.59	42.94	104.21	106.83	99.56	103.22	103.93	100.43
s19	23	303	321	-33.95	-55.73	112.69	102.21	102.30	101.80	103.43	100.85	101.44
s20	146	299	353	54.78	30.96	60.27	107.49	105.35	91.79	106.88	107.46	91.39
s21	860	3014	3411	-108.75	34.43	934.32	110.14	99.52	101.10	110.06	99.10	101.11
Total	**1256**	**12628**	**13002**	**-1767.28**	**-369.02**	**3392.30**	**104.69**	**106.48**	**103.09**	**104.99**	**103.63**	**102.94**

[a] *Values of columns (1-6) measured in thousand of workers.*
[b] *Columns (4-6) do not always add up to the numbers in column (1) due to rounding.*

Column (1) reports the actual change in sectoral labor requirements in the 21 Spanish sectors between 1986 and 1994. Labor requirements increased in all but six sectors, "Agriculture" (s1), "Energy" (s2), "Mining products" (s3), "Chemical products" (s5), "Transport equipment" (s9) and "Textiles" (s11). Columns (4)-(6) present the results of the decomposition analysis applying the PB approach with the additional information considered. The values for the Spanish economy as a whole in the bottom row are obtained by simply adding the sectoral results. Clearly, declining labor requirements per unit of gross output would have led to lower workforce use in most sectors if nothing else would have changed (the $\Delta\mathbf{u}$ effect is generally negative). The generally negative results for the $\Delta\mathbf{L}$ effect suggest that the domestic input coefficients have changed in such a way that labor requirements would have decreased, in the absence of changes in the level and composition of final demand. This result can be given by changes in technology (technological progress or substitution of inputs) or by variations in the trade pattern of Spain. Finally, the contributions of the $\Delta\mathbf{f}$ effect are positive for all sectors. This is not a very surprising result, since in the demand-driven input-output model, positive final demand growth (which has actually happened in the Spanish economy) will always yield increases in labor requirements, unless labor requirements per unit of output are reduced considerably and/or input coefficients cause substitution of inputs towards inputs with lower labor cost coefficients.

Besides the results of the decomposition, the figures of columns (7)-(12) are of most interest. Columns (7)-(9) present ratios ρ^F that compare the effect obtained by PB approach with those obtained by taking average over the 24 traditional decomposition formulae. A value of the ratio equal to 100 implies that the same result would be obtained by both approaches, in other words, the information included does not lead to results remarkably different from a "non-informative situation" where an average solution is the most appropriate solution. Columns (10)-(12) present an equivalent ratio ρ^P for a comparison of the PB technique with the mean of polar decompositions, since this approach is often used because it yields very similar results to the full mean over all decomposition forms with fewer computations being required[19].

As we can see from these columns, the deviations are sometimes considerable. Although in general terms the results obtained by PB approach are quite close to both average decompositions (see the values for the whole of sectors in the last row), for some specific sectors the contributions of the effects vary remarkably. The results for changes in the final demand "Agriculture" (1) are a good example, because if we apply the PB approach to this sector we will obtain that the effect of variations in its final demand is around 18% greater than the result obtained by an average decomposition. "Building materials and construction" (14) or "Textiles" (11) are other examples of great variations for the effects of respectively, changes in labor use per unit of output and Leontief matrix.

Roughly speaking, the most substantial deviations are found for the sectors for which the estimated parameters deviate strongly from one, the value implicitly chosen when taking averages (see Appendix C). However, some exceptions to this generic rule can be found, due to the matricial nature of the decomposition at hand. Consequently, a estimated time path for one of the factors in a sector *i* (let's say the final demand for commodities produced by this sector) that differs greatly from one, can well affect

[19] The reader could feel that is not possible that all the ratios are greater than 100 (see sector 1, for example), since positive and negative deviations have to compensate each other if we add up the three effects. However, note that negative values for some of the effects prevent this. Following with the example of sector 1, the negative effects obtained by the PB approach for the first two effects are greater (in absolute value) to those obtained by average solutions. In addition, the same occurs with the positive effect of the final demand. At the end, we have three ratios grater than 100 but the sum of the effects equals the whole change in the sector.

the contribution of changes in other factor for other sector *j* (for example, the labor requirements per unit of output in sector *j*) although its corresponding estimated path does not differ remarkably form a linear one.

6.2. Comparison with a dynamic average decomposition

The above empirical application illustrates how the PB approach can use limited information of the dependent variable to obtain unique decompositions. Outcomes of Table 1 showed that the results were remarkably different from averages decompositions for some specific cases. This statement suggests that the technique proposed in the paper could be considered as an alternative decomposition methodology to more traditional procedures. Once stated this, the question is: Which one (the PB approach or some of the other methodologies) obtains the most "accurate" outcomes? This subsection is devoted to testing if the results of PB decomposition are somehow more "reliable" than those obtained by using more pragmatic average solutions.

In Section 5 of the present paper, we have already commented the general usefulness of a dynamic decomposition to reduce the variability in the decomposition results. For our case, an annual dynamic decomposition will be computed and used as a benchmark in the analysis. In a scenario with T periods from the initial to the final year, the expressions of the effects obtained by this average dynamic decomposition would be:

$$\mathbf{\Delta u}\text{ Effect} = \sum_{t=1}^{T}\left[(\mathbf{\Delta\hat{u}}_{t-1}^{t})\mathbf{L}^{t-1}\mathbf{f}^{t-1} + \frac{1}{2}(\mathbf{\Delta\hat{u}}_{t-1}^{t})(\mathbf{\Delta L}_{t-1}^{t})\mathbf{f}^{t-1} + \frac{1}{2}(\mathbf{\Delta\hat{u}}_{t-1}^{t})\mathbf{L}^{t-1}(\mathbf{\Delta f}_{t-1}^{t}) + \frac{1}{3}(\mathbf{\Delta\hat{u}}_{t-1}^{t})(\mathbf{\Delta L}_{t-1}^{t})(\mathbf{\Delta f}_{t-1}^{t})\right] \quad (46a)$$

$$\mathbf{\Delta L}\text{ Effect} = \sum_{t=1}^{T}\left[\mathbf{\hat{u}}^{t-1}(\mathbf{\Delta L}_{t-1}^{t})\mathbf{f}^{t-1} + \frac{1}{2}(\mathbf{\Delta\hat{u}}_{t-1}^{t})(\mathbf{\Delta L}_{t-1}^{t})\mathbf{f}^{t-1} + \frac{1}{2}\mathbf{\hat{u}}^{t-1}(\mathbf{\Delta L}_{t-1}^{t})(\mathbf{\Delta f}_{t-1}^{t}) + \frac{1}{3}(\mathbf{\Delta\hat{u}}_{t-1}^{t})(\mathbf{\Delta L}_{t-1}^{t})(\mathbf{\Delta f}_{t-1}^{t})\right] \quad (46b)$$

$$\mathbf{\Delta f}\text{ Effect} = \sum_{t=1}^{T}\left[\mathbf{\hat{u}}^{t-1}\mathbf{L}^{t-1}(\mathbf{\Delta f}_{t-1}^{t}) + \frac{1}{2}(\mathbf{\Delta\hat{u}}_{t-1}^{t})\mathbf{L}^{t-1}(\mathbf{\Delta f}_{t-1}^{t}) + \frac{1}{2}\mathbf{\hat{u}}^{t-1}(\mathbf{\Delta L}_{t-1}^{t})(\mathbf{\Delta f}_{t-1}^{t}) + \frac{1}{3}(\mathbf{\Delta\hat{u}}_{t-1}^{t})(\mathbf{\Delta L}_{t-1}^{t})(\mathbf{\Delta f}_{t-1}^{t})\right] \quad (46c)$$

Such a dynamic decomposition can be computed since the Spanish Statistical Institute provides annual information of the three factors considered in the decomposition problem from 1986 to 1994. Note that this allows dividing the whole period 1986-1994 in 7 stages (1986-1987, 1987-1988, and so on) [20], although an essential issue is that it requires much more additional information than the used by the PB

[20] Note that in the present case, given the data availability, this is the closest situation to the ideal scenario with an infinite number of intermediate points.

approach. Within this dynamic context, the full mean of all decomposition forms has been computed in every stage. Columns (1) to (3) of Table 2 show the effects of the three factors obtained by this dynamic average decomposition.

These figures have been taken as a yardstick to compare the results of the PB and the other average decompositions. The basic idea is that the closer their outcomes are to the dynamic decomposition results, the better a decomposition methodology will perform. So, we will consider as the most accurate decomposition methodology the one that obtains outcomes most similar to this dynamic average decomposition. Table 2 compares the outcomes obtained by the full mean of decompositions, the mean of polar decompositions and the PB approach with these reference results.

Table 2: Comparison of the results with a dynamic decomposition[a]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Sector	***Δu Eff***	***ΔL Eff***	***Δf Eff***	e_u^F	e_L^F	e_f^F	e_u^P	e_L^P	e_f^P	e_u^B	e_L^B	e_f^B
s1	-697.08	-277.84	431.92	7856.63	1303.30	15559.79	8402.16	903.00	14814.12	2004.65	511.23	4540.56
s2	-48.68	-23.94	38.63	49.46	4.09	81.98	53.17	2.26	77.37	34.53	0.81	45.91
s3	-16.01	-45.92	43.94	0.67	8.34	4.29	0.06	2.98	2.22	0.60	0.74	0.01
s4	-39.46	-27.61	69.06	35.76	15.99	99.56	38.42	12.68	95.25	10.17	1.22	18.44
s5	-52.65	-19.86	71.51	0.92	10.26	5.03	0.33	5.92	3.46	2.45	4.74	0.37
s6	-50.52	-20.41	95.93	29.91	5.08	59.63	31.89	3.60	56.91	8.73	0.67	14.23
s7	4.90	-1.04	14.14	18.00	0.33	23.21	18.00	0.33	23.21	17.94	0.38	23.51
s8	-82.86	-12.38	100.24	24.76	4.39	50.00	27.56	2.39	46.20	5.23	1.25	11.59
s9	-177.18	-12.40	169.59	109.15	0.26	120.04	120.12	0.27	109.08	15.80	0.44	10.96
s10	-36.76	-16.10	74.87	0.83	10.29	5.27	0.77	9.84	5.10	5.27	6.23	0.04
s11	-273.57	5.97	187.59	13.41	3.96	31.95	13.23	4.17	32.24	1.36	24.13	14.02
s12	-9.08	-8.01	42.09	0.03	8.03	7.05	0.03	7.92	7.00	0.46	6.11	3.22
s13	50.97	-62.47	34.49	79.99	11.62	30.62	85.83	16.41	27.18	50.38	0.12	55.35
s14	-5.89	-16.92	260.82	0.94	9.41	16.28	0.94	9.38	16.27	0.12	4.39	5.94
s15	-79.13	57.66	357.47	2180.73	6.21	1954.13	2199.14	8.33	1936.78	2959.89	18.30	2512.75
s16	118.02	4.12	65.86	4002.75	0.31	3932.28	4000.54	0.28	3934.47	3504.73	0.43	3427.55
s17	-170.23	22.78	157.45	17.18	2.22	7.05	20.89	5.47	4.98	31.18	5.04	11.15
s18	-29.68	27.11	33.57	84.81	0.12	91.37	91.83	0.16	84.36	117.68	2.19	87.78
s19	-17.69	-73.54	114.23	241.21	363.22	12.44	229.20	333.97	9.83	264.59	317.01	2.37
s20	74.75	30.67	40.59	565.71	1.64	628.29	552.03	3.46	642.88	398.75	0.08	387.25
s21	92.76	42.96	724.28	36673.40	69.94	39946.39	36701.85	67.48	39916.72	40608.02	72.73	44117.96
Total	**-1445.07**	**-427.19**	**3128.26**	**228.00**	**42.88**	**250.33**	**229.32**	**37.42**	**248.69**	**223.70**	**31.28**	**235.14**

[a] *Values of columns (1-3) measured in thousand of workers.*

As commented previously, columns (1)-(3) present the results of the average annual decomposition analysis; again the last row presents the results for the whole of the Spanish economy. The following columns report square differences for each effect of the full mean of decompositions (columns 4–6), the mean of polar decompositions (columns7-9) and the PB approach (columns 10-12), respectively marked with superscripts *F*, *P* and *PB*. The empirical application of the PB methodology described in the previous subsection used only some limited additional information, *i. e.*, sectoral labor use in 1989 and 1992. In contrast, the average decompositions do not require any additional information but the initial and final values of the factors. Note that, the results obtained by the PB technique are the closest to the dynamic decomposition for each of the three effects. The conclusion would be that, using only a limited amount of additional information, one would obtain more "accurate" results than applying more traditional average solutions. Of course, this result applies only to the example taken as illustration, so more empirical exercises have to be made to compare the performance of the PB technique compared with other decomposition methodologies.

7. CONCLUSIONS

A well-known problem of SDA is that the results often depend strongly on the specific decomposition formula chosen, whereas numerous formulae are equivalent from a theoretical point of view. This non-uniqueness problem is often solved rather pragmatically, by reporting an average of (a subset of) all possible formulae. This paper applies a decomposition methodology using Generalized Maximum Entropy (GME) econometrics to select the decomposition formula that provides an optimal 'fit' to additional empirical information. The point of departure is the "Path Based" (PB) method proposed in previous works, showing that one specific solution of this technique is equivalent to taking the average over all traditional decomposition formulae. Since the solutions of this method depend on unknown parameters that characterize the paths of the determinants, these parameters can be estimated even if the available data is very limited. If information about the values of the dependent variable is available for intermediate periods between the initial period and the final period of the analysis, a non-linear GME program can be solved to estimate them and obtain a unique decomposition.

We applied the methodology to quantify the contributions of three determinants of changes in sectoral labor requirements in Spain between 1986 and 1994, i.e. labor requirements per unit of gross output, input coefficients and final demand levels. As additional information, data of sectoral workforce use in 1989 and 1992 were included. The results indicate that, firstly, the use of additional information in the PB approach can well yield results that differ substantially from the mean over all traditional decomposition formulae. Secondly, an annual average decomposition was also obtained to take it as a benchmark for a comparison, showing that the PB approach yielded in this case closer results than more traditional average solutions. These results lead us to believe that the PB method provides an interesting alternative to computing averages over decomposition formulae.

8. REFERENCES

BARFF, R.A. AND P.L. KNIGHT III (1988), "Dynamic Shift-Share Analysis", *Growth and Change*, vol. 19, pp. 2-10.

DE HAAN, M. (2001), "A Structural Decomposition Analysis of Pollution in The Netherlands", *Economic Systems Research*, vol. 13, pp. 181-196.

DIETZENBACHER, E., A.R. HOEN and B. LOS (2000), "Labor Productivity in Western Europe 1975-1985: An Intercountry, Interindustry Analysis", *Journal of Regional Science*, vol. 40, pp. 425-452.

DIETZENBACHER, E., M.L. LAHR and B. LOS (2004), "The Decline in Labor Compensation's Share of GDP: a Structural Decomposition Analysis for the United States, 1982-1997", in: E. Dietzenbacher and M.L. Lahr (eds.), *Wassily Leontief and Input-Output Economics* (Cambridge UK: Cambridge University Press), pp. 138-185.

DIETZENBACHER, E. and B. LOS (1998), "Structural Decomposition Techniques: Sense and Sensitivity", *Economic Systems Research*, vol. 10, pp. 307-323.

DIETZENBACHER, E. and B. LOS (2000), "Structural Decomposition Analyses with Dependent Determinants", *Economic Systems Research*, vol. 12, pp. 497-514.

FERNÁNDEZ, E. (2004), *The Use of Entropy Econometrics in Decomposing Structural Change*, unpublished PhD thesis, University of Oviedo (Spain).

FERNÁNDEZ, E., B. LOS and C. RAMOS (2005), "Using Additional Information in Structural Decomposition Analysis", 15th International Conference on Input-output Techniques, Beijing.

GOLAN, A., G. JUDGE and D. MILLER (1996), *Maximum Entropy Econometrics: Robust Estimation with Limited Data* (Chichester UK, John Wiley).

GOLAN, A., G. JUDGE and S. ROBINSON (1994), "Recovering Information from Incomplete or Partial Multisectoral Economic Data", *Review of Economics and Statistics*, vol. 76, pp. 541-549.

GOLAN, A., J. M. PERLOFF and E. Z. SHEN (2001), "Estimating a Demand System with Non-negative Entries: Mexican Meat Demand", *Review of Economics and Statistics*, vol. 83, pp. 541-550.

GRENDAR, M., JR. and GRENDAR, M. (2004), "Maximum Entropy Method with Non-linear Moment Constraints: Challenges", in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, G. Erickson and Y. Zhai (eds.), AIP (Melville), pp. 97-109.

HARRISON, W.J., J.M. HORRIDGE and K.R. PEARSON (2000), "Decomposing Simulation Results with Respect to Exogenous Shocks", *Computational Economics*, vol. 15, pp. 227-249.

HOEKSTRA, R. and J.C.J.M. VAN DEN BERGH (2003), "Comparing structural and index decomposition analysis", *Energy Economics*, vol. 25, pp. 39-64.

INE (several years), *Contabilidad Nacional de España 1986-1994* (Madrid, Instituto Nacional de Estadística).

INE (several years), *Tablas input-output de España 1986-1994* (Madrid, Instituto Nacional de Estadística).

KAPUR, J.N. and H.K. KESAVAN (1993), *Entropy Optimization Principles with Applications* (New York: Academic Press).

PARIS, Q. and R.E. HOWITT, (1998), "An Analysis of Ill-Posed Production Problems Using Maximum-Entropy", *American Journal of Agricultural Economics*, Vol. 80, pp. 124-138.

ROBINSON, S., A. CATTANEO and M. EL-SAID (2001), "Updating and Estimating a Social Accounting Matrix Using Cross-Entropy Methods", *Economic Systems Research*, vol. 13, pp. 47-64.

ROSE, A. and S. CASLER (1996), "Input-Output Structural Decomposition Analysis: A Critical Appraisal", *Economic Systems Research*, vol. 8, pp. 33-62.

SHANNON, J. (1948), "A Mathematical Theory of Communication", *Bell System Technical Bulletin Journal*, vol. 27, pp. 379-423.

SUN, J.W. (1998), "Changes in Energy Consumption and Energy Intensity: A Complete Decomposition Model", *Energy Economics*, vol. 20, pp. 85-100.

VOGT A. (1978): "Divisia Indices on Different Paths". In W. Eichhorn et al. (eds.), *Theory and Application of Economic Indices*. Physica-Verlag, Wurzburg.

Appendix A: Construction of Data for Empirical Illustration

This appendix depicts the main sources of information consulted for the empirical study in Section 6, as well as the manipulations we had to carry out before we could apply the decomposition analysis to changes in sectoral labor requirements in Spain. The original tables for 1986 and 1994 had a sectoral classification distinguishing 57 sectors. For the sake of simplicity in the paper we have preferred to work with a less detailed classification and we have aggregated them until considering only 21 sectors (see Table A.1). This same aggregation was made with the data of sectoral labor requirements for the initial and final periods, as well as the data used as additional information in 1989 and 1992:

Table A.1: Sector classification

Sector	Name	Equivalence to R57
s1	Agriculture	1
s2	Energy	2+3+4+5+6+7+8+9+10+11
s3	Minerals and mining products	12+13
s4	Non-metallic products	14+15+16+17
s5	Chemical products	18
s6	Metallic products excepting transport equipment	19
s7	Machinery for agriculture and industry	20
s8	Office equipment, measuring equipment and others	21+22
s9	Transport equipment	23+24
s10	Food, drinks and tobacco	25+26+27+28+29
s11	Textiles, leather and clothing	30+31
s12	Paper and derived products	33+34
s13	Industries not elsewhere classified	32+35+36
s14	Building materials and construction	37
s15	Commerce an repairing services	38+39
s16	Restaurants, hotels and cafes	40
s17	Transport services	41+42+43+44+45
s18	Communications	46
s19	Finance and insurance	47+48
s20	Real estate and services to companies	49+50
s21	Other services	51+52+53+54+55+56+57

Appendix B: Estimates of the parameters

The values that appear in the following tables have been obtained using GAMS and the CONOPT2 algorithm to non-linear optimisation problems.

Table B.1: estimates for θ_u and θ_f

Sector	$\hat{\theta}_{uk}$	$\hat{\theta}_{fk}$
s1	0.632	0.000
s2	0.970	0.970
s3	0.635	1.005
s4	1.244	0.802
s5	0.632	0.791
s6	1.385	0.434
s7	0.993	0.897
s8	1.213	0.710
s9	0.999	0.782
s10	0.865	0.459
s11	0.800	0.752
s12	1.252	0.404
s13	0.526	1.309
s14	1.525	0.000
s15	2.723	0.506
s16	1.005	0.000
s17	0.525	0.821
s18	0.814	1.059
s19	0.799	0.996
s20	0.700	0.000
s21	2.903	0.650

Table B.2: estimates for θ_1

$\hat{\theta}_{lkj}$	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13	s14	s15	s16	s17	s18	s19	s20	s21
s1	0.979	0.999	0.999	1.000	0.984	0.999	0.999	0.994	0.971	0.815	0.992	0.995	0.991	0.989	0.992	0.877	0.995	1.000	0.997	0.997	0.974
s2	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	0.997	0.999	0.998	0.999	1.000	1.000	0.999	1.000
s3	0.998	0.997	0.916	0.998	0.991	0.947	0.975	0.945	0.801	0.987	0.997	0.999	0.994	0.874	0.979	0.986	0.990	1.000	0.999	0.990	0.964
s4	1.000	1.001	0.998	0.988	1.010	1.000	1.000	1.002	1.011	1.005	1.002	1.000	1.001	1.100	1.007	1.011	1.002	1.000	1.002	1.010	1.006
s5	0.999	1.000	1.000	1.000	0.990	0.999	1.000	0.997	0.992	0.994	0.978	1.001	1.000	0.987	0.994	0.988	0.999	1.000	0.999	0.998	0.997
s6	0.991	1.002	0.992	0.998	1.003	1.005	1.004	0.999	1.080	0.991	0.973	0.999	1.012	1.044	1.018	1.025	1.010	1.000	1.003	1.008	1.057
s7	0.991	1.001	0.994	0.997	0.996	0.997	1.001	0.991	1.004	0.990	0.999	0.999	0.998	1.004	1.001	1.002	1.001	1.000	1.000	1.003	1.048
s8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.003	1.006	1.000	1.000	1.000	1.000	1.002	1.001	1.000	1.001	1.000	1.000	1.000	1.004
s9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	0.999	0.995	0.999	0.995	1.000	1.000	0.999	0.989
s10	1.012	1.000	1.000	1.000	0.999	1.000	1.000	1.000	0.999	1.004	0.994	1.000	0.999	0.999	0.999	0.953	1.000	1.000	1.000	1.000	1.000
s11	1.000	1.000	1.000	1.000	1.001	1.000	1.000	1.000	1.000	1.002	0.990	1.000	1.001	1.009	1.001	1.000	1.000	1.000	1.000	1.000	1.001
s12	0.987	1.003	0.997	0.995	0.980	0.998	0.995	1.000	1.009	0.949	0.994	1.071	1.010	1.026	1.054	1.032	1.002	1.006	1.020	1.024	1.001
s13	0.996	1.005	1.000	1.002	1.017	1.007	0.998	1.008	1.094	1.019	1.041	1.001	1.085	1.164	1.035	1.028	1.002	1.001	1.008	1.015	1.041
s14	0.926	1.021	0.979	0.970	0.927	0.985	0.980	0.987	0.965	0.864	1.001	0.980	1.019	1.026	1.323	1.408	1.094	1.009	1.216	1.715	0.926
s15	0.941	0.998	0.911	0.982	0.967	0.971	0.982	0.971	0.993	0.818	0.967	0.982	1.001	1.010	0.997	1.022	0.957	1.002	1.009	1.005	0.989
s16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.001	0.999	1.001	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.002
s17	1.006	0.999	1.001	1.001	1.002	1.001	1.001	1.001	1.002	1.010	1.003	1.001	1.000	0.999	1.000	0.997	1.008	0.999	0.999	0.999	1.006
s18	1.003	1.001	1.001	1.002	1.006	1.002	1.002	1.003	1.007	1.009	1.006	1.001	1.002	1.010	1.010	1.013	1.006	1.001	0.997	1.008	1.027
s19	1.006	0.997	1.001	1.001	1.004	0.999	0.999	0.998	1.000	1.004	0.994	1.001	1.000	0.935	0.981	0.968	0.998	1.000	0.844	1.008	1.006
s20	0.990	0.998	0.996	0.997	0.986	0.995	0.994	0.990	0.966	0.972	0.989	0.996	0.996	0.970	0.997	0.992	0.995	1.001	1.010	0.991	0.981
s21	0.997	1.000	0.996	0.997	0.973	0.997	0.997	0.980	0.991	0.990	0.996	0.998	0.996	1.006	1.010	1.002	0.987	1.004	1.010	1.175	0.754