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Holding Period Return-Risk Modeling: The Importance of Dividends

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ABSTRACT

In this paper we explore the relevance of dividends in the total equity return over longer time horizons. In addition, we investigate the effects of different reinvestment assumptions of dividends. We use a unique set of revised and corrected US equity data series, comprising monthly prices and dividends based on consistent definitions over the period 1871-2002 (132 years). Our findings are relevant for performance evaluation, for estimating the historical equity risk premium, and for investment simulation.

Keywords: dividends, holding period return, geometric mean.

Un modelo rentabilidad riesgo para la inversión en activos: la importancia de los dividendos

RESUMEN

En este trabajo se estudia la relevancia de los dividendos como componente del rendimiento de los activos financiero en el horizonte del largo plazo. Adicionalmente, se estudian varias alternativas de reinversión para estos dividendos. Se usaran series de datos procedentes del mercado americano con información sobre precios y dividendos para el periodo comprendido entre 1871 y 2002. Los resultados son relevantes de cara al estudio de la rentabilidad, de la estimación de la prima de riesgo así como para la simulación de distintas alternativas de inversión.

Palabras Clave: Dividendos, Rendimiento, Media Geométrica.

Clasificación JEL: C13, C22, C89, G14.

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1. INTRODUCTION

In this paper we explore the relevance of dividend return as part of total equity return.

Since the seminal paper of Rozeff [1984], the predictability of equity returns by dividend yields has been researched intensively (see Campbell & Shiller [1988a,b], Goetzmann & Jorion [1995] and Goetzmann, Ibbotson & Peng [2001], e.g.), and the findings are somewhat poor and mixed at least. We investigate the effects of different reinvestment assumptions of dividends and analyze the importance of dividends in the total holding period return over longer time horizons. We use a unique set of revised and corrected US equity data series, comprising monthly prices and dividends based on consistent definitions over the period 1871-2002 (132 years). This long history enables us to avoid overlapping bias when estimating risk and return statistics for longer holding periods. In many empirical studies the sample period starts in 1926 (Dimson, Marsh & Staunton [2004] being a notable exception), and our data set allows us to analyze potential differences between the pre- and post-1926 periods. Our findings are relevant for investment simulation (cf. Freeman [1992]), performance evaluation and for estimating the market risk premium.

This paper is organized as follows. Section 2 introduces notation and summarizes various return definitions. Section 3 discusses and describes the data set. Section 4 is devoted to the relevance of dividends and reinvestment assumptions, and investigates some investment strategies. Section 5 summarizes and concludes the paper.

2. NOTATION AND DEFINITIONS

Discretely and continuously compounded returns

We introduce the following notation and definitions. We start from the price index *PI* which represents an equity price series adjusted for stock splits and stock dividends. The discretely compounded price return (capital appreciation or «price relative») p_t over the period t is given by:

$$1 + p_t = \frac{PI_t}{PI_{t-1}} \qquad \textbf{[1]}$$

where PI_t and PI_{t-1} denote the price index at the end of period t and $t-1$, respectively. Hence the appreciation of the price index over the period (t,T) is:

$$\frac{PI_T}{PI_t} = \prod_{\tau=1}^{T-t} \left(1 + p_{t+\tau}\right) \qquad \textbf{[2]}$$

The discretely compounded dividend yield y_t over the period t is:

$$y_t = \frac{D_t}{PI_{t-1}} \qquad \textbf{[3]}$$

where D_t is the cash dividend paid at the end of period t. Combining [1] and [3], the discretely compounded total return (or «value relative») over the period t, r_t, is given by:

$$1 + r_t = 1 + p_t + y_t =: \left(1 + p_t\right)\left(1 + d_t\right) \qquad \textbf{[4]}$$

where d_t denotes the dividend ratio:

$$d_t = \frac{y_t}{1 + p_t} = y_t \frac{PI_{t-1}}{PI_t} = \frac{D_t}{PI_t} \qquad \textbf{[5]}$$

The dividend ratio relates the dividend to the stock price at the payment date, not to the previous price. The dividend ratio (and not the dividend yield) will prove to be relevant when considering continuous compounded returns and when analyzing the dividend reinvestment effect over some holding period $[0,T]$.[1]

The cumulative total returns equity index with periodic reinvestment of dividends, *TRI*, is defined by:[2]

[1] Note that one plus the dividend yield is the arithmetic difference between the value and the price relatives, whereas one plus the dividend ratio is the geometric difference (i.e. ratio) between the value and the price relatives.

[2] Without loss of generality, the starting values of the price index and the total returns index can be scaled to obtain $PI_0 = TRI_0$, or even set to unity: $PI_0 = TRI_0 = 1$. The latter holds anyway when t=0 is the inception date of the equity series.

$$TRI_T = TRI_t \prod_{\tau=1}^{T-t}\left(1+r_{t+\tau}\right) = TRI_t\left[\prod_{\tau=1}^{T-t}\left(1+p_{t+\tau}\right)\right]\left[\prod_{\tau=1}^{T-t}\left(1+d_{t+\tau}\right)\right] \quad [6]$$

In addition to equity, we consider a risk-free investment opportunity (Treasury Bills, e.g.). The discretely compounded risk-free rate over the period *t* is given by b_t. The cumulative risk-free returns index, *BI*, is:

$$BI_T = BI_t \prod_{\tau=1}^{T-t}\left(1+b_{t+\tau}\right) \quad [7]$$

The excess return on equities is defined as $r_t - b_t$. When the equities are representative for the stock market as a whole, the market risk premium is the expected excess return, $E\left\{r_t - b_t\right\}$.[3]

The continuously compounded return is obtained by taking the natural logarithm of one plus the discretely compounded return. Using [4], the continuously compounded total equity return over the period *t* is $\ln\left(1+r_t\right) = \ln\left(1+p_t\right) + \ln\left(1+d_t\right)$.Considering the dividend ratio instead of the dividend yield allows us to express the log total return as the sum of the log price return and the log dividend ratio.[4]

Arithmetic and geometric mean returns

When studying returns over a long horizon, the compounded average growth rate or geometric mean return becomes relevant. The geometric mean of the returns $\left\{\tilde{r}_t\right\}_{t\in T}$ over *T* periods, $G\left(\tilde{r}_t;T\right)$, is defined as:

$$G\left(\tilde{r}_t;T\right) = \left[\prod_{t=1}^{T}\left(1+\tilde{r}_t\right)\right]^{1/T} - 1 \quad [8]$$

[3] Since the risk premium is the return on a self-financing portfolio, it does not make sense to distinguish between a nominal and a real risk premium. After all, expected inflation is contained in the equity return as well as in the risk-free rate. Another way to see this is to consider a portfolio of x in equities and 1-x in risk-free assets. The nominal portfolio return is $r_{p,t} = x\left(r_t - b_t\right) + b_t$, where expected inflation is contained in the risk-free rate.

[4] This approach is not to be confused with the dividend ratio model developed by Campbell & Shiller [1988a,b]; see also Campbell, Lo & MacKinlay [1997]. Since they want to model log dividend growth, they approximate the log of the sum of price and dividend with a weighted average of log price and log dividend.

When returns are intertemporally independent and identically distributed, then (according to the strong law of large numbers) the geometric mean converges almost surely to the constant $\exp\left[E\left\{\ln\left(1+\tilde{r}_t\right)\right\}\right]-1$ as $T \to \infty$. This implies that the distribution of the geometric mean degenerates and converges to a point distribution.

In practice, this limit is not reached. However, when T is sufficiently large and when the stationarity and independence assumptions are satisfied, the distribution of the logarithm of one plus the geometric mean is approximately normal with mean μ and variance σ^2/T, where μ and σ^2 are the mean and variance of the log returns $\ln\left(1+\tilde{r}_t\right)$. Hence, the distribution of the geometric mean is approximately lognormal with mean $\exp\left[\mu+½\sigma^2/T\right]-1$ and variance $\exp\left[2\mu+\sigma^2/T\right]\left\{\exp\left[\sigma^2/T\right]-1\right\}$; see Michaud [1981], e.g. Note that a confidence interval of the log of one plus the geometric mean will be symmetric, whereas confidence intervals of the geometric mean and the end-of-horizon value will be asymmetric. For large T, this lognormal distribution has properties similar to those of a normal distribution; hence the expected value approaches the median.

The geometric mean is the rate of return that compounds initial value V_0 to T-period terminal value V_T: $V_T = V_0\left[1+G\left(\tilde{r}_t;T\right)\right]^T$. Given the previous results, it follows that the asymptotic distribution of T-period terminal value is lognormal. The a-quantiles of the distributions of final value and geometric mean are related by $V_T^{(\alpha)} = V_0\left[1+G^{(\alpha)}\left(\tilde{r}_t;T\right)\right]^T$. Since for large T the expected geometric mean approaches the median value, the expected geometric mean relates to the median terminal value, $V_T^{(0.5)} = V_0\left[1+E\left\{G\left(\tilde{r}_t;T\right)\right\}\right]^T$. The arithmetic mean m of the discretely compounded returns, in contrast, relates to the expected terminal value, $E\left\{V_T\right\} = V_0\left[1+m\right]^T$, from which we recognize the familiar valuation maxim. Since $E\left\{G\left(\tilde{r}_t;T\right)\right\} < m$ for $\sigma^2 > 0$, the median of the final value distribution is lower than the mean, indeed implying a right-skewed distribution.

Given the mean m and the variance s^2 of the discretely compounded returns, a very accurate approximation to the geometric mean can be obtained through:[5]

$$G\left(\tilde{r}_t;T\right) \approx (1+m)\exp\left[-\frac{½s^2}{\left(1+m\right)^2}\right]-1 \qquad [9]$$

This approximation (actually, all of the derived approximations) clearly reveals the «variance slippage»: the negative relationship between the geometric mean and the variance of returns.

3. DATA AND DESCRIPTIVE STATISTICS

The data set runs from December 31, 1871 through December 31, 2002.[6] We use a unique set of revised and corrected US equity data series, comprising monthly prices and dividends based on consistent definitions over the period 1871 through 2002 (132 years). These data are based on the S&P500 Index and Cowles's extensions as described in Wilson & Jones [1987, 2002]. All prices are measured ultimo month except for the sub-period 1871:01 through 1885:02, for which only mid-month prices are available. However, important is that prices are not averaged over each month. Compared to other available data sets this is a distinguishing feature; it is well known that the use of within-month averaged prices generates various statistical biases in the return series.[7] From the monthly prices, a price index is constructed. Monthly dividends were estimated from trailing quarterly dividends by Wilson & Jones [2002] and used to construct a cumulative total returns index with monthly reinvestment.

As a proxy for the risk-free rate we use the monthly total return on US Treasury Bills. Since T-Bills were only introduced in 1929, the risk-free rate series consists from 1870:12 through 1912:12 of 75% of the commercial paper yield, and from then on until 1928:12 of the yield on short-term government bonds.

We have divided the total sample period in various sub-periods. Since 1926 is the base year of the S&P Indexes (i.e. the S&P90 and from 1957:03 on the familiar S&P500) we consider 1871-1925 and 1926-2002. The period 1963-2002 is consistent with an evaluation horizon of 40 years. To allow putting recent developments in a broader historical context, we finally set a breakpoint at 1983.

Table 1 presents some descriptive statistics over various sub-periods, obtained from monthly return data. We have annualized the means and medians by simply multiplying monthly figures with 12. In this way, the average price return and average

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[5] See Michaud [1981] and Jean & Helms [1983] for a comparison of various approximations and further references to the literature. When arithmetic means are estimated with sampling error, a bias correction on the geometric mean must be applied; see Blume [1974], Jacquier, Kane & Marcus [2003] and Hallerbach [2003b]. We do not pursue this issue here.

[6] I thank Jack Wilson (College of Management at North Carolina State University, Raleigh NC 27695, wilson@gw.fis.NCSU.EDU) for generously providing me with the equity and T-Bill data sets.

[7] See for example Schwert [1990], Wilson, Jones & Lundstrum [2001] and Hallerbach [2003a].

dividend yield sum to the average total return. All return series exhibit excess kurtosis and to some lesser degree skewness. A Jarque-Bera test rejects normality for all series and all (sub-) periods at p=0.0000, except for the risk-free return over the most recent sub-period 1983-2002 (p=0.085). Comparing means and medians we see that the distributions of price returns, total returns and excess returns are skewed to the left, except for the period 1963-1982.[8] The distributions of the dividend yield and the dividend ratio (and to a lesser extent the T-Bill return), in contrast, are skewed to the right. This can be explained by the fact that these return figures are restricted to non-negative values.

Over the total sample period the total return on stocks was on average almost 10% p.a. with an annualized volatility of 16.7% (monthly volatility times $\sqrt{12}$). The period after 1926 shows both a higher average return and a higher standard deviation. However, although the mean returns over 1926-1962 and 1963-2002 are almost the same, the volatility is substantially higher in the first sub-period.

The average excess return is an estimate of the annualized historical *monthly* equity risk premium, since the risk-free rate is measured over the same interval as the stock returns. Over the full 132 years it equals about 6% p.a. Over the most recent 40-year period it is about 5% p.a. where the risk premium of about 7.5% over the most recent 20 years sharply contrasts with the 2.5% over the period 1963-1982. In the latter period the average total return on stocks is 80 basis points below the overall period average whereas at the same time the average risk-free rate reached its historical high.

Instead of annualizing monthly means and standard deviations, it seems an appealing alternative to estimate annual statistics directly from annual return series. However, this not only reduces the number of observations (thus increasing sampling error) but also raises the complex issue of temporal return aggregation. In Hallerbach [2003b] we show that holding period risk and return statistics exhibit an extraordinary sensitivity to the choice of the starting point in calendar time.

Comparing their statistics, the dividend yield and the dividend ratio are almost identical. The level of the average dividend yield / ratio has declined steadily over time. This seems consistent with Fama & French [2001] who argue that the propensity to pay cash dividends has declined over time. This is refuted by DeAngelo, DeAngelo & Skinner [2004] who show that aggregate dividends paid actually increased over the

[8] This is confirmed by visual inspection of the empirical frequency distributions. All skewness statistics, however, are positive. This is caused by some extreme observations in the right tails. Hence the positive skewness suggested by the positive third moment is not real but apparent. Indeed, a zero third order moment is a necessary and not a sufficient condition for distributional symmetry and knowledge of the third moment gives almost no clue about the shape of the distribution; see Mood, Graybill & Boes [1974, pp.75-76].

last decades. However, we note that at the same time the level of the average price return has increased, most markedly over the last 20 years. Since the dividend yield is a function of both dividends and stock prices, dividend yields can also decrease because of increasing prices. Indeed, the level of S&P500 dividends has increased steadily over time, until September 2000 when stock prices started plunging and the dividend level stabilized.

The standard deviation of the dividend yield / ratio is very low, comparable to the volatility of the risk-free rate over time. Although the dividend yield does contribute its share to the total stock return, it does not contribute to the volatility: the standard deviations of price and total stock returns are virtually the same. Even when average total return remains the same, decreasing dividend yield (and hence increasing average price return) implies that a larger portion of the total return is subject to risk. We further explore the importance of dividends in the next section.

Table 2 shows correlations between the return series. The almost perfect positive correlation between price return, total return and excess return is not surprising. After all, the volatilities of the dividend yield and the risk-free rate are low. In addition, the very weak correlations between price return on the one hand and dividend yield and risk-free rate on the other indicate large diversification effects within the total return and the excess return. The correlation between the dividend ratio and the price return tends to be negative, except for the period 1983-2002. This follows directly from the correlation between the dividend yield and the price return and the reciprocal relationship between dividend ratio and price return. The latter relationship also explains the negative correlation between the dividend ratio and the total stock return. Most striking is the correlation between the risk-free rate and the dividend yield / ratio. Before 1926 it is negative and after 1926 it turns to positive. Over the most recent 40 and 20 years it increases substantially to 24% and 45%, respectively.

Table 3 displays the annualized arithmetic mean and standard deviation of the price and value relatives. In contrast to Table 1, the means are now compounded to *per annum* figures.[9] In addition, the actual geometric mean return according to eq.[8], its approximation eq.[9] and its 95% confidence interval is provided. The effect of variance slippage is pronounced for the price return and total return, and almost absent for the dividend ratio and the risk-free rate. The geometric mean approximation according to eq.[9] is outstanding. For the price and total returns, the 95% confidence interval is quite wide, even for the overall period of 132 years. One dollar invested in the stock market in January 1871, with dividends reinvested, has grown to $(1.0879)^{132}$ = \$ 67,679 in December 2002; this is the median horizon value. The expected horizon value was a

[9] Under the simple annualization used in Table 1, the artifact can arise that the arithmetic mean p.a. is smaller than the geometric mean p.a.

staggering $(1.1031)^{132}$ = \$ 419,765 and the difference with the median value clearly indicates how skewed the distribution of horizon value is. The 95% confidence interval of horizon value is between a modest $(1.0574)^{132}$ = \$ 1,578 and an astounding $(1.1193)^{132}$ = \$ 2,902,588 … For some periods, the confidence interval of the geometric mean price return extends to negative compound growth rates, but for the total return the confidence interval is strictly positive.

Striking is that the overall period geometric means of the price return and the risk-free rate are almost the same. This implies that the end value obtained by a roll-over strategy of one-month risk-free Bills from 1871:12 on was approximately equal to the cumulative price return obtained in the stock market. Stated otherwise: the equity risk premium was fully generated by the (reinvested) cash dividends. Figure 1 plots the total return Bill index *BI* and the stock price index *PI* over time. Many empirical studies start their sample in 1926, but there are fundamental differences between the pre- and post-1926 periods. Comparing the geometric means in Table 3, we see that in the period 1871-1925 the largest part of the equity return was generated by (reinvested) dividends, whereas in the period 1926-2002 the importance of dividends has decreased and the contribution of the risky price return to the total return was higher. Jones, Wilson & Lundstrum [2002] also discuss this trade-off of price return and dividend return. Finally note that in the 1926-2002 period, the 95% confidence intervals of the total equity return and the risk-free rate do not overlap; at this confidence level, equities «dominate» T-Bills.

4. DIVIDENDS AND REINVESTMENT ASSUMPTIONS

Figure 1 clearly illustrates the importance of dividends in the total return on stocks. Although the ratio of dividend return to price return has decreased over time, dividends account for 42% of the geometric mean total return p.a. over the period 1926-2002. Over longer horizons the compounding effect kicks in and the differences between a price index and total return index increase. Most available stock market indexes are capital appreciation indexes; hence they offer an incomplete picture of the stock market's performance. For this reason, Clarke & Statman [2000] disqualify the DJIA and S&P500 indexes as investment benchmarks.

In this section we further analyze the proportion of total holding period return that can be allocated to (reinvested) dividends as well as the effect of different reinvestment assumptions.

Proportion reinvested dividends

The total return index as defined by eq.[6] shows that the dividends are reinvested at the total return r_t. The question arises what part of the total return, realized over some holding period, is due to the dividends and their reinvestment value. A straightforward way to obtain the proportion reinvested dividends of the total return realized over the holding period $[0,T]$ is to simply consider the differences between the accumulated price returns and total returns. Using eqs.[2] and [6], the amount of reinvested dividends as a fraction $f(r;T)$ of total accumulated investment value at the end of the holding period (i.e. initial investment plus total return) is:

$$f(r;T) = 1 - \frac{PI_T/PI_0}{TRI_T/TRI_0} = 1 - \prod_{t=1}^{T}\left(\frac{1+p_t}{1+r_t}\right) = 1 - \left[\prod_{t=1}^{T}(1+d_t)\right]^{-1} \quad \textbf{[10]}$$
$$= 1 - \left[1 + G(d_t;T)\right]^{-T}$$

When cash dividends are paid, $d_t \geq 0$ for all t and $d_t > 0$ most of the time. When the volatility of the dividend ratio is not too large, then $G(d_t;T) > 0$. So when the horizon is long enough, reinvested dividends will *always* dominate the total horizon value. In the limit, when $T \to \infty$, $f(r;T) \to 1$.

Eq.[10] is a simple expression. However, in order to be able to investigate the effects of alternative reinvestment assumptions later on, we have to adopt an alternative approach. At the end of each period t the cash dividend $D_t = y_t \cdot PI_{t-1} = d_t \cdot PI_t$ is received. This is invested over the remaining holding period $[t,T]$ at the equity's total rate of return. So the horizon value (the future value at time T) of the period t reinvested dividend is $y_t\, PI_{t-1}\left(TRI_T/TRI_t\right)$. Summing all reinvested dividends over the periods $[1,T]$ yields an index (dollar) amount of:

$$\sum_{t=1}^{T} y_t\, PI_{t-1}\frac{TRI_T}{TRI_t} = PI_0\sum_{t=1}^{T}\left(y_t\frac{PI_{t-1}}{PI_t}\right)\frac{PI_t}{PI_0}\frac{TRI_T}{TRI_t} = PI_0\sum_{t=1}^{T} d_t\frac{PI_t}{PI_0}\frac{TRI_T}{TRI_t} \quad \textbf{[11]}$$

This is the horizon value of the reinvested dividends emanating from investing the amount of $PI_0 = TRI_0$ in equities at $t=0$. Over the holding period, the price index itself grows from PI_0 to PI_T. So the total value of capital appreciation plus all reinvested dividends at the end of the holding period is:

$$PI_T\left[1+\sum_{t=1}^{T} d_t\left(\frac{TRI_T/TRI_t}{PI_T/PI_t}\right)\right] = PI_T\left[1+\sum_{t=1}^{T} d_t\left(\prod_{\tau=1}^{T-t}\left(1+d_{t+\tau}\right)\right)\right] \equiv TRI_T \quad \textbf{[12]}$$

Hence, the amount of reinvested dividends as a fraction of total accumulated investment value at the end of the holding period is:

$$f(r;T) = 1 - \frac{PI_T / PI_0}{TRI_T / TRI_0} = 1 - \left[1 + \sum_{t=1}^{T} d_t \left(\prod_{\tau=1}^{T-t} (1 + d_{t+\tau}) \right) \right]^{-1} \qquad [13]$$

Eqs.[10] and[13]are equivalent. The proof that indeed $1 + \sum_{t=1}^{T} d_t \left(\prod_{\tau=1}^{T-t} (1 + d_{t+\tau}) \right)$ $= \prod_{t=1}^{T} (1 + d_t)$ for all T follows by induction and is left to the reader.

Table 4 shows $f(r;T)$ for our sample periods. For the total period, reinvested dividends account for not less than 99.74% of total horizon value. Even for the period 1983-2002 where the mean dividend ratio is only 20% of total mean return p.a., reinvested dividends still account for almost 43% of total accumulated investment wealth. Since $f(r;T)$ depends on the length of the horizon, we also report this fraction based on the geometric mean dividend ratio from other periods. Even for the low dividend ratio observed over the past 20 years, the largest part of total horizon value (thus including initial investment value) is generated by reinvested dividends. Although the volatility of the dividend ratio is low, it can interact with future total returns. The correlation between the dividend ratio and subsequent total returns (at which dividends are reinvested) will have some effect on total reinvestment value. This brings us to considering alternative reinvestment strategies.

The effect of different dividend reinvestment assumptions

Now suppose that the dividends received are not reinvested in the equity index, but instead at the short-term risk-free rate. Under this reinvestment assumption, the sum of all reinvested dividends in eq.[11] changes to:

$$\sum_{t=1}^{T} y_t \, PI_{t-1} \frac{BI_T}{BI_t} = PI_0 \sum_{t=1}^{T} d_t \frac{PI_t}{PI_0} \frac{BI_T}{BI_t} \qquad [14]$$

In addition, the price returns accumulate from P_0 to P_T. Hence, the total value of capital appreciation plus all risk-free reinvested dividends at the end of the holding period, TRI'_T, now becomes:

$$TRI'_T = PI_T \left[1 + \sum_{t=1}^{T} d_t \frac{BI_T / BI_t}{PI_T / PI_t} \right] = PI_T \left[1 + \sum_{t=1}^{T} d_t \left(\prod_{\tau=1}^{T-t} \left(\frac{1 + b_{t+\tau}}{1 + p_{t+\tau}} \right) \right) \right] \qquad [15]$$

Analogous to eq.[13] , the amount of dividends reinvested at the risk-free rate b_t, as a fraction of total accumulated investment value at the end of the holding period is:

$$f(b;T) = 1 - \frac{PI_T/PI_0}{TRI'_T/TRI_0} = 1 - \left[1 + \sum_{t=1}^{T} d_t \left(\prod_{\tau=1}^{T-t} \left(\frac{1+b_{t+\tau}}{1+p_{t+\tau}} \right) \right) \right]^{-1} \qquad \textbf{[16]}$$

Table 5 shows selected statistics, including $f(b;T)$, for the investment strategy in which all cash dividends are reinvested at the one-month T-Bill rate. Of course, for longer horizons the one-month T-Bill return is no longer risk-free and this roll-over strategy will generate reinvestment risk. Comparing geometric means it is striking that the pre- and post-1926 periods are so similar. The same applies to the 1963-1982 and 1983-2002 periods, even though for these periods the ratio of dividend return and price return is so different. It seems that with incurring only limited risk, the total return over the risk-free rate can be enhanced by investing in equities but «safe-guarding» the dividends by reinvesting in T-Bills. The actual dividend return will determine the monthly inflow to the T-Bill account.

However, reinvesting 100% of the dividends in T-Bills is an extreme strategy (with reinvesting 100% of dividends in equities at the other extreme). We therefore investigated a strategy in which dividends are put into a separate investment account. A constant fraction of this account is invested monthly in T-Bills, the remaining part plowed back into equities. The «optimal» fraction is determined by maximizing the Sharpe [1966,1994] ratio, defined as the ratio of mean and standard deviation of excess returns. Table 6 contains the results. In all periods except 1963-1982 and 1983-2002, it would have been optimal (with hindsight) to short T-Bills and reinvest more than 100% of dividends in equities – which is not feasible. For the two most recent 20-year periods, however, the risk-return trade-off would have improved by re-investing part of dividends at the risk-free rate.[10] The results of this experiment are limited but call for further research.

As a starting point it makes sense to consider equities as a package of a price return and a dividend return generator. However, since not only the higher expected return but also the higher risk stem from the price return component it may make sense to repackage equities by reinvesting part of dividends in other assets than equities. The performance of the mixed reinvestment strategy depends on the correlation between

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[10] For the sake of completeness we note that there is actually a band around the specified reinvestment fractions which yields approximately the same Sharpe ratio. This implies that approximately the same risk-return trade-off could have been obtained by dividing the total investment portfolio over equities and T-Bills.

the dividend ratio and subsequent total equity returns and T-Bill returns. This relationship, as well as the relationship between cash dividends and dividend return (or ratio), is worth investing further. Finally, since dividend return is virtually risk-free when compared to price return, disentangling total equity return may shed a better light on changes in total equity risk and return over time.

5. SUMMARY AND CONCLUSIONS

We investigated the effects of different reinvestment assumptions of dividends and uncovered the profound importance of dividends in the total return over longer time horizons. We found that a roll-over strategy with T-Bills outperformed the equity price index over the total period 1871-2002. This implies that the equity risk premium was fully generated by reinvested dividends! In many empirical studies the sample period starts in 1926 but we find that the pre- and post-1926 periods are markedly different when focusing on the proportion of price return and dividend return in the total return.

Our empirical results confirm the intuition that for equities not only the higher expected return but also the higher risk stem from the price return component. This in turn suggests to consider equities as a package of a price return and a dividend return generator. Pursuing this line of reasoning, it may make sense to repackage equities by reinvesting part of dividends in other assets than equities. In one of the reinvestment strategies we here considered, all cash dividends are reinvested at the risk-free rate (roll-over T-Bills). Although in the post-1926 period the average price return is more than 2.5 times as large as in the pre-1926 period, this dividend reinvestment strategy generated comparable (arithmetic and geometric) mean returns. Even more surprising is that the dividend reinvestment strategy yielded virtually identical (arithmetic and geometric) mean returns over the last four decades: especially in this sub-period relatively safe dividend return has been traded for risky price return (the price return almost doubled and the dividend yield decreased with more than 30%). Forming investment strategies in which the price returns and dividend returns are re-packaged may be another route along which the intertemporal relationship between dividend returns, total returns, and risk-free rates may be uncovered. In addition, since dividend return is virtually risk-free when compared to price return, disentangling total equity return may shed a better light on changes in total equity risk and return over time. This is a challenging route for further research.

Table 1: Descriptive statistics.

Mean, median and standard deviation of monthly discretely compounded price returns, dividend returns, dividend ratios, total returns and excess returns on stocks, and of T-Bill returns, expressed in percent per annum. Means / medians and standard deviations are annualized simply by multiplying monthly figures with 12 and $\sqrt{12}$, respectively. Normality is rejected (Jarque-Bera) for all periods at the p=0.0000 level, except for the risk-free T-Bill return over the period 1983-2002 (p=0.085).

in %		stocks					T-Bill
		price return	dividend yield	dividend ratio	total return	excess return	total return
1871-2002 :	mean :	5.31	4.53	4.53	9.85	5.92	3.93
(132 yrs)	median :	7.05	3.97	3.96	12.15	7.80	3.69
	st.dev. :	16.73	0.77	0.78	16.71	16.72	0.82
1871-1925 :	mean :	2.78	5.14	5.14	7.91	4.02	3.89
(55 yrs)	median :	3.38	4.88	4.83	9.00	5.12	3.76
	st.dev. :	13.03	0.71	0.71	13.02	13.00	0.51
1926-2002 :	mean :	7.13	4.10	4.10	11.23	7.27	3.96
(77 yrs)	median :	10.68	3.11	3.08	15.03	11.03	3.58
	st.dev. :	18.93	0.80	0.81	18.91	18.93	0.99
1926-1962 :	mean :	6.46	4.86	4.87	11.32	9.84	1.48
(37 yrs)	median :	11.18	4.34	4.37	16.95	15.70	1.04
	st.dev. :	22.38	0.88	0.90	22.35	22.35	0.43
1963-2002 :	mean :	7.75	3.40	3.39	11.15	4.89	6.25
(40 yrs)	median :	9.97	2.64	2.65	12.80	7.35	5.60
	st.dev. :	15.06	0.64	0.64	15.07	15.08	0.89
1963-1982 :	mean :	5.06	3.99	3.99	9.05	2.36	6.69
(20 yrs)	median :	3.58	2.94	2.94	6.85	2.67	5.60
	st.dev. :	14.52	0.77	0.77	14.50	14.53	1.08
1983-2002 :	mean :	10.43	2.81	2.78	13.24	7.42	5.82
(20 yrs)	median :	13.08	2.52	2.52	16.21	11.35	5.60
	st.dev. :	15.58	0.43	0.42	15.63	15.60	0.63

Table 2: Correlations.

Correlations among monthly discretely compounded price returns, dividend returns, dividend ratios, total returns and excess returns on stocks, and T-Bill returns. For each period, the number of months T is given, as well as the critical values of the correlation coefficient at the 95% (bold typeface) and 99% (bold italic) confidence levels, respectively.

	price return	dividend yield	dividend ratio	total return	excess return	price return	dividend yield	dividend ratio	total return	excess return
						1871-2002 (T=1584; **0.0493**, ***0.0647***) :				
div. yield						-0.0455				
div. ratio						***-0.1316***	***0.9950***			
total return						***0.9989***	0.0008	***-0.0856***		
exc. return						***0.9976***	0.0032	***-0.0829***	***0.9988***	
TB return						0.0228	**-0.0494**	**-0.0547**	0.0205	-0.0287
	1871-1925 (T=660; **0.0763**, ***0.1002***) :					1926-2002 (T=924; **0.0645**, ***0.0847***) :				
div. yield	-0.0384					-0.0399				
div. ratio	***-0.1189***	***0.9961***				***-0.1316***	***0.9941***			
total return	***0.9985***	0.0158	-0.0649			***0.9991***	0.0022	***-0.0898***		
exc. return	***0.9980***	0.0112	-0.0694	***0.9992***		***0.9975***	0.0076	***-0.0841***	***0.9986***	
TB return	0.0609	***0.1180***	***0.1107***	0.0673	0.0282	0.0128	***-0.1043***	***-0.1088***	0.0084	-0.0437
	1926-1962 (T=444; **0.0931**, ***0.1221***) :					1963-2002 (T=480; **0.0895**, ***0.1175***) :				
div. yield	-0.0542					-0.0095				
div. ratio	***-0.1643***	***0.9919***				-0.0753	***0.9968***			
total return	***0.9992***	-0.0147	-0.1252			***0.9991***	0.0332	-0.0327		
exc. return	***0.9989***	-0.0115	-0.1220	***0.9998***		***0.9981***	0.0147	-0.0509	***0.9983***	
TB return	0.0090	***-0.1632***	***-0.1653***	0.0025	-0.0165	0.0101	***0.3119***	***0.3068***	0.0234	-0.0358
	1963-1982 (T=240; **0.1267**, ***0.1660***) :					1983-2002 (T=240; **0.1267**, ***0.1660***) :				
div. yield	-0.0559					0.1057				
div. ratio	-0.1216	***0.9969***				0.0255	***0.9957***			
total return	***0.9986***	-0.0031	-0.0691			***0.9996***	**0.1327**	0.0526		
exc. return	***0.9968***	-0.0206	-0.0859	***0.9972***		***0.9993***	0.1148	0.0346	***0.9992***	
TB return	-0.0081	***0.2352***	***0.2291***	0.0043	-0.0698	0.0600	***0.4481***	***0.4470***	0.0721	0.0316

Table 3: Arithmetic and geometric mean returns.

Annualized arithmetic mean m and standard deviation s of monthly discretely com-pounded returns. One plus the arithmetic mean is annualized by exponentiating to the power 12; the standard deviation is annualized by multiplying with $\sqrt{12}$. $G(\cdot;T)$ is the actual geometric mean according to eq.[8]. $G(\cdot;T)$ H» denotes the approximation by eq.[9] on the basis of monthly data. Both the geometric mean and the approximation are annualized by exponentiating to the power 12. «95% confid.» indicates the 95% confidence interval of the geometric mean.

%		price return	dividend ratio	total return	T-Bill return
1871-2002 :	m	5.45	4.63	10.31	4.00
(132 yrs)	s	16.73	0.78	16.71	0.82
	$G(\cdot;T)$	3.98	4.62	8.79	4.00
	$G(\cdot;T)^{\sim}$	3.99	4.62	8.80	4.00
	95% confid.	1.05 7.00	4.48 4.76	5.74 11.93	3.86 4.15
1871-1925 :	m	2.81	5.26	8.21	3.96
(55 yrs)	s	13.03	0.71	13.02	0.51
	$G(\cdot;T)$	1.94	5.26	7.30	3.96
	$G(\cdot;T)^{\sim}$	1.95	5.26	7.31	3.96
	95% confid.	-1.53 5.53	5.06 5.45	3.66 11.06	3.82 4.10
1926-2002 :	m	7.36	4.18	11.83	4.03
(77 yrs)	s	18.93	0.81	18.91	0.99
	$G(\cdot;T)$	5.47	4.17	9.87	4.03
	$G(\cdot;T)^{\sim}$	5.48	4.17	9.88	4.03
	95% confid.	1.09 10.03	3.99 4.36	5.34 14.60	3.80 4.26
1926-1962 :	m	6.65	4.98	11.93	1.49
(37 yrs)	s	22.38	0.90	22.35	0.43
	$G(\cdot;T)$	4.04	4.98	9.21	1.49
	$G(\cdot;T)^{\sim}$	4.04	4.97	9.22	1.49
	95% confid.	-3.20 11.82	4.67 5.28	1.66 17.33	1.35 1.63
1963-2002 :	m	8.03	3.44	11.74	6.44
(40 yrs)	s	15.06	0.64	15.07	0.89
	$G(\cdot;T)$	6.81	3.44	10.48	6.43
	$G(\cdot;T)^{\sim}$	6.82	3.44	10.50	6.43
	95% confid.	1.91 11.94	3.23 3.64	5.43 15.78	6.14 6.72
1963-1982 :	m	5.18	4.06	9.44	6.90
(20 yrs)	s	14.52	0.77	14.50	1.08
	$G(\cdot;T)$	4.09	4.06	8.32	6.89
	$G(\cdot;T)^{\sim}$	4.08	4.06	8.31	6.89
	95% confid.	-2.30 10.90	3.71 4.41	1.70 15.37	6.39 7.39
1983-2002 :	m	10.95	2.82	14.08	5.98
(20 yrs)	s	15.58	0.42	15.63	0.63
	$G(\cdot;T)$	9.60	2.82	12.69	5.98
	$G(\cdot;T)^{\sim}$	9.63	2.82	12.72	5.98
	95% confid.	2.28 17.45	2.63 3.01	5.15 20.77	5.68 6.27

Table 4: Proportion $f(r;T)$ of end-of-horizon investment value generated by dividends reinvested at the total equity return (in percent).

The column heading specifies the period from which the input data are taken. The row heading indicates the period over which the fraction reinvestment value is evaluated, according to eq.[10].

% evaluation period :	input from period : 1871-2002	1871-1925	1926-2002	1926-1962	1963-2002	1963-1982	1983-2002
1871-2002 :	**99.74**	99.88	99.55	99.84	98.85	99.48	97.45
1871-1925 :	91.68	**94.03**	89.45	93.08	84.42	88.80	78.32
1926-2002 :	96.92	98.07	**95.71**	97.62	92.59	95.33	88.24
1926-1962 :	81.22	84.98	77.97	**83.41**	71.37	77.07	64.25
1963-2002 :	83.60	87.12	80.52	85.66	**74.13**	79.65	67.11
1963-1982 :	59.50	64.11	55.86	62.13	49.13	**54.89**	42.65
1983-2002 :	59.50	64.11	55.86	62.13	49.13	54.89	**42.65**

Table 5: Cash dividends reinvested monthly at the risk-free rate (total T-Bill return).

Annualized arithmetic mean, median and standard deviation of monthly discretely com-pounded returns. One plus the arithmetic mean (median) is annualized by exponentiating to the power 12; the standard deviation is annualized by multiplying with $\sqrt{12}$. $G(\cdot;T)$ is the actual geometric mean according to eq.[8], annualized by exponentiating to the power 12.

$f(b;T)$ indicates the fraction of total end-of-horizon investment value generated by cash dividends reinvested monthly at the T-Bill rate. All in percent.

in %	period : 1871-2002	1871-1925	1926-2002	1926-1962	1963-2002	1963-1982	1983-2002
mean	5.21	5.14	5.26	2.99	7.40	7.38	7.42
median	5.80	5.42	6.03	3.65	7.92	7.25	8.39
st.dev.	4.33	5.11	3.68	3.53	3.72	3.42	4.00
$G(\cdot;T)$	5.11	5.01	5.19	2.93	7.32	7.32	7.33
$f(b;T)$	75.99	80.42	63.50	44.26	51.22	54.25	30.79

Table 6: Comparison of investment strategy in which optimal part of dividends is reinvested monthly at risk-free rate (maximizing the Sharpe ratio), with roll-over T-Bill investment strategy, full dividend reinvestment at T-Bill rate and a full equity (re-)investment strategy.

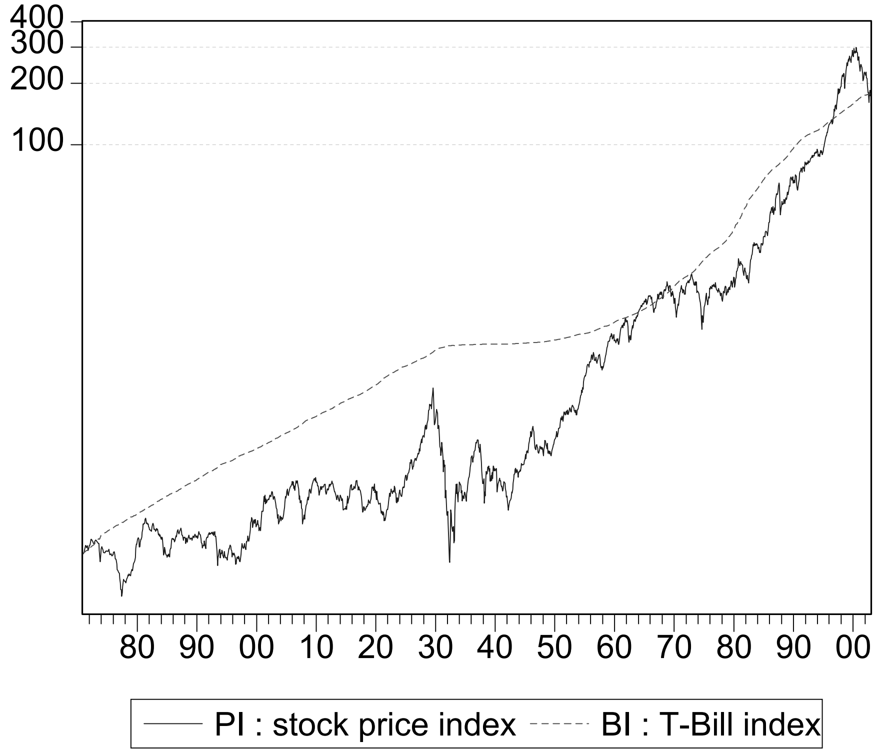
Annualized arithmetic means, medians and standard deviations of monthly discretely com-pounded returns. One plus the arithmetic mean (median) is annualized by exponentiating to the power 12; the standard deviation is annualized by multiplying with $\sqrt{12}$. $G(\cdot;T)$ is the actual geometric mean according to eq.[8], annualized by exponentiating to the power 12.

The optimal part of dividends reinvested at the risk-free rate is determined by maximizing the Sharpe ratio (the quotient of mean and standard deviation of monthly excess returns). All in percent.

		100% T-Bill investment	100% of dividend reinvested at T-Bill rate	part of dividend reinvested at T-Bill rate: % reinv at T-Bill rate	part of dividend reinvested at T-Bill rate: total portf. return	100% equity investment
1963-1982 :	mean	6.90	7.38	35.35	8.55	9.44
	median	5.75	7.25		6.76	7.07
	st.dev.	1.08	3.42		9.49	14.50
	$G(\cdot;T)$	6.89	7.32		8.07	8.32
1983-2002 :	mean	5.98	7.42	11.74	13.10	14.08
	median	5.75	8.39		16.28	17.48
	st.dev.	0.63	4.00		13.81	15.63
	$G(\cdot;T)$	5.98	7.33		12.03	12.69

Figure 1: Stock price index and T-Bill index.

Stock price index *PI* is the S&P500 price index, dividends excluded. The T-Bill index *BI* is the total return index from a roll-over strategy in one-month T-Bills. The series start ultimo 1871:12 at 1.00 (logscale) and ends ultimo 2002:12.



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