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Alternative approaches to regional convergence exploiting both spatial and temporal information

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ABSTRACT

The standard approaches used in the empirical literature to test economic convergence-divergence between countries and regions are all grounded on the Mankiw-Romer-Weil and Barro-Sala-i-Martin contributions that led to the celebrated β-convergence model. Such a model, however, presents strong limitations. This paper reviews some of the approaches proposed in the literature that seek to overcome these limitations and aim to capture the full dynamics of the economic convergence process. Four approaches are reviewed. The first is based on the theory of space-time processes, the second is a spatial versions of panel data modelling, the third is grounded on a spatially adjusted continuous time specification and the fourth on the concept of stochastic convergence as it was developed in the time series literature.

Keywords: Regional convergence, Stochastic convergence, Spatial panel data models, Unit-roots; Systems of differential equations. Spatial Economics.

Aproximaciones alternativas a la convergencia regional utilizando tanto información especial y temporal

RESUMEN

Las aproximaciones estándar utilizadas en la literatura empírica para contrastar la convergencia-divergencia económica entre los países y regiones están todas relacionadas con las contribuciones de Mankiw-Romer-Weil y Barro-Sala-i-Martin que llevan al celebrado modelo de b convergencia. Tal modelo, sin embargo, presenta fuertes limitaciones.Este papel revisa algunas de las aproximaciones propuestas en la literatura que buscan superar estas limitaciones y que tienen por objetivo capturar las dinámicas completas del proceso de convergencia económico. Se revisan cuatro aproximaciones. La primera está basada en la teoría de procesos espacio-temporales, la segunda es una versión espacial de la modelización de datos panel, la tercera se basa en una especificación temporal continua ajustada espacialmente y la cuarta en el concepto de convergencia estocástica, tal y como se ha desarrollado en la literatura de series temporales.

Palabras clave: Convergencia regional, Convergencia estocástica, Modelos de datos panel espaciales, Raíces unitarias, Sistemas de ecuaciones diferenciales, economía espacial.

Clasificación JEL: C31, C33, R11, R10.

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1. INTRODUCTION

The most popular approaches to study the regional convergence of per-capita income are all stemming from the neo-classical Solow-Swan (Solow, 1956; Swan, 1956) model of long run growth and by the framework developed from by Mankiw et al. (1992) and Barro and Sala-i-Martin (1995). This framework led to the now celebrated b-convergence approach, an empirically testable model that seeks to identify convergence by verifying the inverse relationship between the growth in per-capita income at a certain moment of time and the income level at the beginning of the time period. The β-convergence model, therefore, is not a dynamic model *strictu sensu*, but a model based on the comparison between two time periods.

This is a major drawback under both the theoretical and the applied point of view. In fact an economist is usually interested in studying the full dynamics of the convergence process, that is the path followed by per-capita incomes in the various regions in the whole period considered. Indeed very different situations may lead to the same results in terms of the β-convergence (see Figure 1) and this *equifinality* of different models may cause problems in the phase of result interpretation and its use in political decisions and targeting resources.

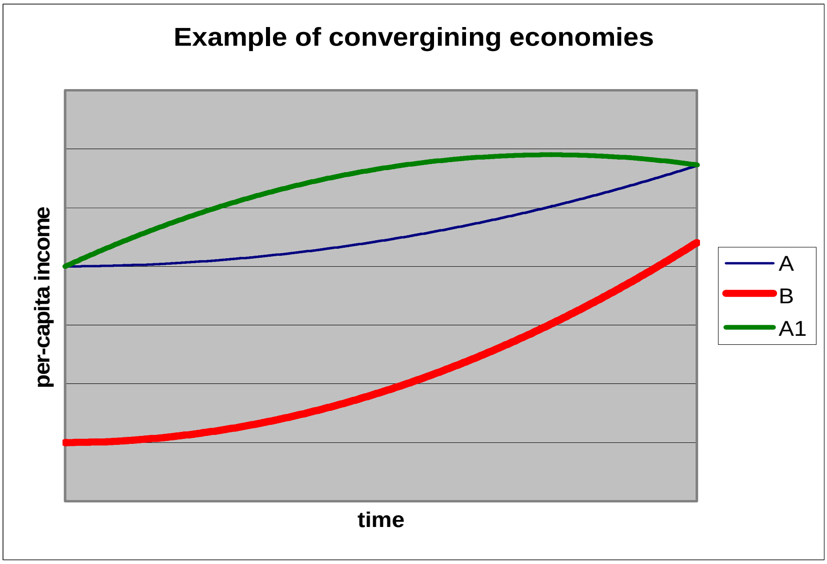

Figure 1: Example of the equifinality of the β-convergence model. Economies A and B β-converge at exactly the same speed than economies A1 and B even if the two temporal patterns are quite different. Indeed economies A and B converge along the whole period, whereas A1 and B diverge in the first years and then rapidly converge in the very last years of the period considered.

This paper reviews some of the approaches proposed in the literature to overcome this problem and to capture the full dynamics of the convergence process.
The paper is organized as follows. Section 2 contains a review of the more general methodologies related to the modelling of space-time data. Section 3, 4 and 5 concentrate on some of the alternatives models that have been the specific concern of some of my recent empirical studies and are grounded respectively on the Lotka-Volterra continuous time approach (treated in Arbia and Paelink, 2003; 2004), on the panel data modelling (discussed in Arbia and Piras, 2004) and on the stochastic convergence framework (adopted in Arbia and Costantini, 2004). In these sections I will review the theory and some of the empirical findings obtained and I will discuss some of the major methodological problems. Finally Section 6 contains some concluding remarks and directions for further developments in the field.

2. SPACE-TIME STATISTICAL MODELS

The problem of considering simultaneously both spatial and temporal dependence present in the empirical observations is a general one, has an old tradition in the statistical literature, and is by no means typical of regional convergence. The basis for the space-time modelling were set in the seventies through the seminal contributions of Pfeiffer and Deutsch (1980) and Bennett (1979) that introduced the class of space-time autoregressive and moving average processes (STARMA) by extending the general framework proposed by Box and Jenkins (1970) for purely time processes (the ARMA class) in their celebrated book. These processes still represent the point of departure of more complicated conceptualisations.

Let us assume that we observe a random variable (i = 1,.2, …,n; t = 1, 2, …, T) in, say, n regions over T periods of time. A space-time random field Yi,t, i S, t T with S and T appropriate space and time sets, belongs to the Space Time Autoregressive Moving Average class (STARMA; Pfeiffer and Deutsch, 1980; Upton and Fingleton, 1985) if it satisfies the following stochastic differences equation:

$$Y_{it} = \alpha Y_{it-h} + \beta \sum_{i \neq j} w^{(k)}{}_{ij} Y_{it-h} + \gamma u_{it-l} + \lambda \sum_{i \neq j} w^{(k)}{}_{ij} u_{it-l} + u_{it} \qquad \textbf{[1]}$$

with $\alpha, \beta, \gamma, \lambda$ parameters to be estimated, $w^{(k)}{}_{ij} \in W^{(k)}$, $W^{(k)}$ an appropriate connectivity matrix of spatial order k, $\{u_{i,t}, i \in S, t \in T\}$ a spatial white noise field (Arbia, 2005) and h and l the temporal maximum lags. The statistical properties and the estimation and testing procedures associated with this conceptualization are discussed thoroughly in Bennett (1979). The identification phase follows the line of the classical ARMA modelling through the definition of a space-time autocorrelation function that helps in identifying the most significant spatial and temporal lags. The model has found

found many applications, but, until recently, they were mainly concentrated in epidemiology and diffusion processes (Cliff et al, 1975).

Equation (1) can be easlily adapted to accommodate explicative variables leading to the so-called STARMAR (Space Time Autoregressive Moving Average with additional Regression terms) class of models (Upton and Fingleton, 1985) that can be expressed in the form;

$$Y_{it} = \alpha Y_{it-1} + \beta \sum_{i \neq j} w_{ij} Y_{it-1} + \gamma u_{it-1} + \lambda \sum_{i \neq j} w_{ij} u_{it-1} + \phi \mathbf{X}_{t-1} + u_{it} \qquad \textbf{[2]}$$

with **X** a vector of independent variables. Further extensions and thorough references may be found in Bennett (1981) and Hepple (1981).

It is only in the last years that an application of this framework may be found in the applied economic literature. Pace et al. (1998) analysed the real estate market concentrating, in particular on pricing. They noticed that an optimal way of incorporating both spatial and temporal dependencies into empirically feasible pricing models does not seem quite obvious. To better capture the effect of both spatial and temporal information on real estate prices, overcoming the problems associated with indicator variable models, they introduce a model of the STARMA class which uses information from nearby, recently sold, properties in predicting the value of a given property. In other words, instead of assuming that each region has its own effect modelled by a separate parameter, the STAR formulation assumes that nearby properties have the same relationships to the observations across the entire sample. Using data on housing prices they show the substantial benefits obtained by modelling the spatial as well as the temporal dependence of the data. In particular, the spatio-temporal autoregression reduced significatively the median absolute error with reference to an indicator-based model. The improved performances of their specification is confirmed by the analysis of one step-ahead forecast.

A recent contribution to spatio-temporal modelling within the applied econometric literature has been made by the nobel prize Clive Granger together with Giacomini (Giacomini and Granger, 2003) who compared the relative efficiency of different methods of forecasting aggregate space- time economic series obeying to the STARMA family.

Within the context of regional convergence there are no examples so far. The naïve application of this class of models would imply the use of Equation (2) (of which the statistical properties are known) by setting the annual level of per-capita income as the explicative variable X and the one-year growth as the dependent variable Y. However the full economic-theoretic implications of this formalization still need to be clarified.

Under the methodological point of view it should be noted that for years the spatio-temporal modelling has not moved substantially from the STARMA paradigm set out in the eighties. Recently, however, some important extensions were introduced in the literature especially to deal with non-stationarity (a common feature of spatio-temporal data). Under this respect, an important alternative recently suggested in the literature to models (1) and (2) is based on the idea of *non-separable* covariance structure (Cressie and Huang, 1999; Gneiting, 2002). A separable covariance structure is a covariance in which the temporal and the spatial component can be separated and, hence, more easily modelled. Conversely non-separable covariances are more complex to treat. The most popular approaches are obtained by imposing a temporally varying structure, or applying the Fourier approach or, finally, using completely monotonic functions (see Bruno et. al, 2003. For different alternatives see Zhang et. al, 2002). Non-parametric and Bayesian approaches were also exploited by Sampson and Guttorp (1992) and Damian et al. (2001).

3. A SPATIAL PANEL DATA CONCEPTUALISATION

An alternative way of modelling the spatio-temporal variations of great interest in regional convergence analysis is the one grounded on panel data literature. As it is well known panel data allow the contemporaneous study of the dynamic and the individual variation of economic phenomena. Baltagi (2001) lists some of the benefits and of the limitations of using such data (see also Hsiao, 1986; Klevmarken, 1989; Solon, 1989). First of all they allow controlling for individuals heterogeneity. Furthermore, they are more informative than pure time series or purely cross-sectional data, they present more variability, less collinearity among the variables and more degrees of freedom. On the other side of the coin design and data collection problems are more complicated then in the case of pure time series or cross-sectional data. Measurement errors may also arise and may produce distortions in inference. In many instances the time dimension is too short to allow a proper dynamic modelling due to the heavy costs associated with data collection. Finally there are major problems associated with selectivity of the sample arising in the various forms of self-selectivity, non-response, attrition or new entry.

Notwithstanding these problems the diffusion of panel data has been supported by the increasing data availability. Up to only few years ago, the diffusion of panel data sets was restricted to the case of United States, the only country in which panel data were collected on a regular basis. Nowadays, many of the European countries have their own longitudinal surveys (e. g. the Italian Survey on Households Income and Wealth run by the Bank of Italy), and the European Community Household Panel (ECHP) is a precious source of information in empirical economic studies. Spatially referenced panel data are also increasingly popular in economics.

In recent time there has been a wide diffusion of contributions in the statistical methods designed to analyse panel data. However, only a few numbers of papers may be found in the literature dealing with spatial panel data (remarkable exceptions being, e. g., Anselin, 1988, 2001; Kapoor et al, 2003; Anselin et al., 2004).

Some advances have been made in considering prediction in panel data regression models by accounting for spatial autocorrelation among states and regions. Baltagi and Li (1999) derive the best linear unbiased predictor for the random error component model with spatial correlation and compare the performances of several predictors of a simple demand equation for cigarettes based on a panel of 46 states over the period 1963-1992. The estimators they compare in the forecasting exercise are the OLS with fixed effect (both accounting for and disregarding spatial correlation effects) and the GLS estimator for random effect (again both in the case we ignore or we consider spatial correlation effects). The main result obtained is that it is important to take into account spatial correlation and heterogeneity across states because their consideration improve sensibly the performances in terms of RMSE of the forecasts. Baltagi et al. (2003) provide further results and an extension of the previous findings.

Of particular relevance in this respect are the contribution made by Paul Elhorst (2001, 2003). In his works the author offers an exhaustive treatment of the specification of a series of models, elaborated starting from the classical framework of the traditional panel data specification conjugated with the typical forms of modelling spatial dependence. In particular Elhorst elaborates the specification and estimation strategies for spatial panel data models that include spatial error autocorrelation and spatially lagged dependent variable. The author starts from the classical literature on panel data, and adapt what can be learned from the spatial econometric literature by discussing four models: the spatial fixed effect model, the spatial random effect model, and the fixed and random coefficient spatial error models. He also derives the relative likelihood for each model and discusses the asymptotic properties and the estimation procedures. The problems that may arise from the spatial version of these four models are also discussed into detail. Another interesting aspect is the derivation of the likelihood function of a fixed effect dynamic panel data model extended to include spatial error autocorrelation or spatially lagged dependent variables.

In a series of very recent papers (Arbia and Piras, 2004; Arbia Basile Piras, 2004; Arbia Elhorst Piras, 2005, Arbia Basile Piras, 2005) we criticized the use of cross-section and panel data within the context of regional economic convergence and we proposed the use of the framework elaborated by Elhorst (2001, 2003). We also produced the first empirical application of spatial panel data models to European regional convergence.

The basis our criticism is that both cross-sectional regression and fixed effects panel data estimates are characterized by the imposition of strong a-priori restrictions on the parameters that do not fit well when attacking the problems connected with regional economic convergence.

In particular, the main drawbacks of cross-sectional studies concern the complete homogeneity in the parameters describing the growth process. This is a very restrictive hypothesis due to the large technological and institutional gaps between countries that make it more reasonable to assume the existence of significant differences both in the initial conditions and in the rate of convergence. Another important drawback of cross-sectional studies when focusing on regional convergence has to do with the presence of omitted regional-specific, time-invariant variables: the effects connected with these variables that are not explicitly considered in the model are captured by the presence of the fixed-effect in the panel specification.

On the other hand even if it is true that panel data allow for regional heterogeneity, differences across regions are only limited to differences in the intercept term of the model. Thus all regions present a common growth rate (incorporated in the coefficient), but their own starting point may be very different. Differences in starting points concern not only disparities in the initial level of the per-capita income, but also differences in the structural characteristics of the economies, and differences in the initial endowments of factors influencing the growth process. A further problem when using panel data is that the annual growth rate of the per-capita GDP is used as a dependent variable in the empirical analysis. However, as it is remarked in Barro and Sala-i-Martin (1995), regional growth is a long run dynamic phenomenon, and the annual growth rate describes more a particular movement towards a trend rather than a true growth dynamics[1] .

For the above reasons panel data estimates are often considered more reliable than those based on purely cross-sections. However it should not be neglected the fact that, specifically with spatial panel, the standard estimation procedures can be invalid due to the presence of spatial dependence and spatial heterogeneity that may lead to serious biases and inefficiencies in the estimates of the convergence rate.

In Arbia and Piras (2004) we make use of data on per-capita income drawn from the Cambridge Econometrics European Regional database[2].Observed in 125 NUTS2 European regions belonging to 10 European Countries[3] observed in a time period that

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[1] A naive way to solve this problem is to make use of a modified variable like, e. g. a moving average (of five or more years) of the growth rate.

[2] Many empirical works in the convergence literature make use of the REGIO database (like, e. g. Quah, 1996; Baumont, Ertur and LeGallo, 2002; Arbia and Paelink, 2004, amongst the others). However, REGIO presents some critical points. First of all, the data quality is very variable across countries and across time, and, furthermore, the time series presents many missing observations at the NUTS2 level. Moreover, data are expressed in current prices, and there is a considerable delay in the release of new regional data by the National Statistical Institutes of the various countries. For these reasons the authors made the choice of using the Cambridge Econometrics dataset which is an elaboration of the REGIO database (for great detail see European Regional Prospect, 2003).

[3] Belgium, Denmark, France, Germany, Luxembourg, Italy, Netherlands, Portugal, and Spain.

spans from 1977 to 2002. In the paper we start by estimating the classical ß-convergence model. The estimated coefficient, significantly negative, shows the presence of absolute ß-convergence among the 125 European Region. This result is then compared with a model that includes a spatial effect in the form of a spatial error and a spatial lag specification. In both specifications this addition provides a lower convergence rate, a results that is in line with the main literature (Arbia, Basile and Salvatore, 2003). Finally we estimate the rate of convergence using panel data. In particular we considered the following three specifications:

(i) the fixed effect model, expressed in the following form:

$$\Delta Y_{i\text{-}t} = \alpha_i + \beta Y_{i,t\text{-}1} + \varepsilon_{i\text{-}t} \qquad [3]$$

with $\Delta Y_{i\text{-}t}$ the yearly growth rate of per -capita GDP for region *i*, $Y_{i,t_{n-1}}$ the long of per-capita GDP in region *i* at time t-1, α_i a vector of random "country specific" effects assume to be independent and $\varepsilon_{i,t}$ a spatial white noise.

(ii) the fixed effect spatial error model expressed in the following form:

$$\Delta Y_{i,t} = \alpha_i + \beta Y_{i,t\text{-}1} + \varepsilon_{i\text{-}t} \qquad [4]$$

with the error component being distributed according to the Simultaneous Autoregressive (SAR) spatial field (Arbia, 2005), that is

$$\varepsilon_{i,t} = \delta W \varepsilon_{i,t} + \varphi_{i,t} \qquad [5]$$

where, in addition to the previous notation, **W** is a spatial weight matrix and δ .the error component spatial autocorrelation coefficient and $\varphi_{i,t}$ a spatial white noise field.

(iii) the fixed effect spatial Lag Model expressed formally as:

$$\Delta Y_{i,t} = \alpha_i + \lambda \sum_{j=1}^{n} w_{ij} \Delta Y_{i\text{-}t} + \beta Y_{i,t\text{-}1} + \varepsilon_{i\text{-}t} \qquad [6]$$

where, in addition to the previous notation, λ is the spatial correlation coefficient for the variable $\sum_{j=1}^{n} w_{ij} \Delta Y_{i-t}$.

In the empirical analysis contained in the paper the presence of convergence is confirmed in all the specification considered, and, again, the convergence rate is lower when considering the effect of spatial dependence. Thus considering spatial dependence is important in evaluating the convergence process among European regions. Moreover, the spatial lag specification appears to be more reliable and seems to fit better the data.

In the paper we also proposed a further improvement in terms of the model by considering the following specification

$$\Delta Y_{i,t} = \beta_1 Y_{i,t} + \beta_2 W Y_{i,t} + \beta_3 Y_{i,t-1} + \phi X_{i,t} + \mu_i + \eta_t + \varepsilon_{t,t} \qquad [7]$$

which incorporates a time fixed effect, a spatial fixed effect a time lag, a spatial lag and a space-time lagged effect on the dependent variable.

Another point raised in Arbia and Piras (2004) concerns the testing of the hypothesis of independence among residuals in a spatial panel data model. There are two obvious (although partial) approaches that can be followed. The first concerns the test of spatial autocorrelation in the T different moments of time using the classical Moran'I or LM tests (Anselin, 1988). The second refers to the test of temporal autocorrelation in the n locations considered and thus involves the computation of n distinct Durbin-Watson tests (Davidson and MacKinnon, 1993).

A possible way of building a general procedure to test simultaneously the two features could be obtained in the following way. Let us start from the familiar Moran's I expression that (as it is known) is more general and admits the Durbin-Watson procedure as a particular case (see e. g. Arbia, 2005). The general expression is:

$$I = h(\hat{\mathbf{e}}'\hat{\mathbf{e}})^{-1}(\hat{\mathbf{e}}'\mathbf{W}\hat{\mathbf{e}}) \qquad [8]$$

where $\hat{\mathbf{e}}$ are the regression residuals, and h a normalizing factor such that $h = \frac{n}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}$, with $w_{ij} \in \mathbf{W}$. In the case of a cross-section regressions, the dimension of the matrix **W** is n-by-n, where n corresponds to the number of the spatial units considered. Conversely in the case of a panel regression the vector of residuals has a different dimension with respect to the spatial weight matrix. In this respect it is sufficient to build the weight matrix in a block diagonal form with the traditional spatial weight matrix repeated *T* times on the main diagonal. Formally the new space-time connectivity matrix **Ω** can be expressed as

$$\Omega = \begin{pmatrix} W & 0 & \dots & \dots & 0 \\ 0 & W & & & \dots \\ \dots & & W & & \dots \\ \dots & & & \dots & \\ 0 & & & & W \end{pmatrix} \qquad [9]$$

where **W** are n-by-n connectivity matrices. The dimension of the **Ω** matrix is now nT-by-nT, as each block has dimension n-by-n, and the number of blocks corresponds to the number of time periods. The computation of the Moran's I follows straightforwardly by replacing the **W** matrix in Equation (8) with the **Ω** matrix of Equation (9) and stacking the n-by-T matrix of space-time residuals in one single NT-by-1 column vector.

The asymptotic distribution for the Moran statistics, derived under the null hypothesis of no spatial dependence, is still normal as in the classical (purely spatial) formulation. However the expected value and the variance need to be derived explicitly in this situation.

The previous expression accounts for spatial correlation in each time period. In those cases where the model considers both spatial and serial autocorrelation, the structure of the spatial weights matrix is different. In particular, the blocks above and below the main diagonal are also non-zero and the number of diagonals that are different from zero depends on the time periods considered in the serial autocorrelation term. For instance by limiting ourselves to lag1 temporal dependence we have:

$$\Omega = \begin{pmatrix} W & W & \dots & \dots & 0 \\ W & W & W & 0 & \dots \\ \dots & W & W & W & \dots \\ \dots & & & \dots & 0 \\ 0 & & 0 & W & W \end{pmatrix} \qquad [10]$$

that allows for simultaneous spatial and temporal (lag1) correlation amongst residuals to be detected. Alternative approaches have been proposed by Anselin et al. (2004) for the LM test in spatial lag and spatial error panel data models and by Pesaran (2004b) for a diagnostic test for unspecified spatial dependence in panels.

4. THE LOTKA-VOLTERRA CONTINUOUS TIME FRAMEWORK

Traditional convergence analysis provides indications on the convergence of regions towards common steady-states, but not on the time path they followed to reach such steady-state. Starting from this consideration in Arbia and Paelinck (2003; 2004) we analysed the problem using a continuous-time framework (Bergstrom, 1990; Gandolfo, 1990) based on the Lotka-Volterra predator-prey system of non-linear equations (Lotka, 1956 and Volterra reprinted in Chapman, 1931), a model firstly proposed by Samuelson (1971) in an economic context.

In particular Arbia and Paelinck (2003) we considered the case of a panel of, say, n regions observed over T periods of time for which the regional per-capita incomes ($i = 1,.2, ...,n; t = 1, 2, ..., T$) are observed. For each region i at time t the growth rate, $\dot{y}_{it}$, can be expressed as a function of the level of per-capita income y_{it} at time t (as suggested by the classical β-convergence literature), but is also affected by the average level of per-capita incomes in the first order contiguous regions (say y^{*}_{it}), and, possibly, in regions non-contiguous to the first order (say y^{**}_{it}).

Considering all the R regions simultaneously leads to the following system of differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln y_t = \hat{a}y_t + \hat{b}y_t^{*} + \hat{c}y_t^{**} + h \qquad [11]$$

with $y^{*}_{it} = \sum_{i=1}^{n} w^{*}_{ij} y_{it}$; $y^{**}_{it} = \sum_{s=1}^{n} w^{**}_{ij} y_{it}$, $w^{*}_{ij} \in W^{*}$ is the generic element of a (possibly row standardized) first order contiguity matrix and $w^{**}_{ij} \in W^{**}$ the generic element of the complementary (also possibly row standardized) matrix of all higher contiguity orders. The logic on which such a model is grounded is that in practical circumstances spatially specific economic conditions can be important as contextual factors in the explanation of the growth process. For instance, the fact of being surrounded by rich regions can determine a faster growth rate or maybe a slower one (in this latter case the richer region "preys on" the less developed one). The introduction of surrounding growth rates could also be considered.

The terms b_r and c_r in Equation (11) thus represent the spatial interaction terms and can be positive or negative, thus describing various combinations of spatial interaction. Quite obviously, if $b_r = c_r = 0$, model (11) represents a way to estimating the classical β-convergence model in continuous time as it was originally formulated by Barro and Sala-i-Martin (1995) starting from Ramsey (1928) model, and thus without incurring in the problems connected with its discretization commonly used in the applied literature to apply the standard estimation methods.

In Equation (11) various combinations of the parameters may produce a convergence (in a mathematical sense) to a stable singular point, whereas other combinations may push the system to diverge. The conditions for convergence to a stable point are derived from the non-positiveness of the time-derivative of the associated Liapunov function, the negativeness of the real part of the eigenvalues of matrix A implying asymptotic stability (see Braun, 1975; Peschel and Mende, 1986; Gandolfo, 1996, Hahn, 1963).

The concept of economic regional convergence implied by the previous Equation (11) obviously does not coincide with the one usually considered in the literature. The mere fact that the *n* regions satisfy the (mathematical) convergence requirements does not necessarily imply long run equality of per-capita income, because each region is free to follow its own trajectory, possibly leading to *n* distinct convergence paths. Indeed the classical β-convergence models (and their spatial-conditional versions) are finite difference models explaining the net variation of the (log) per-capita income observed in a certain time period and producing summary parameters for the area as a whole, whereas the Lotka-Volterra modelling framework is a system of differential equations describing, for each region, a different convergence path and a different steady-state level. The important feature of this second approach is that it also provides, as a summary, the conditions under which the long-term equilibrium may occur in the entire area.

The advantages of this approach are evident. Rather then averaging in one single parameter (or more if we use one of the spatial-conditional versions) situations that may be very different from one region to the other, it allows a separate modelling for each region. It is possible indeed that, while local (in terms of a time period) observations show a trend towards absolute convergence, an interregional spatial model would embed long term divergent forces, specific to long-term histories of rise and decay of individual regions. This describes a particular interpretation of the *spatially conditional* convergence.

An application of the extension of this model was presented in Arbia e Paelinck (2004). In this second paper we considered the dynamics of per-capita income in 119 NUTS2 European Regions in the years 1985-1999 estimating Equation (11) using Simultaneous Dynamic Least Squares (Paelinck, 1996). We obtained empirical evidence that European regions do not converge to a common value of their per-capita income even if the mathematical system of equation does (*mathematically)* convergence to a stable point. Moreover, the remarkable result of the conditional convergence for all

119 regions is observed. In other word all regions follow their own growth paths and converge to their own steady-states.

To interpret the empirical results obtained, we contrasted the outcomes of our model with those of a classical Barro and Sala-i-Martin model and with its spatially corrected counter-part. The three models were estimated with reference to the same dataset referring to the 119 European NUTS2 regions in the period 1980-1994. The three models provided consistent results in that the estimates of the fundamental convergence parameter are always negative in the classical model and in its spatial version and are also so in all regions when using the Lotka-Volterra specification. A problem, however, that was not treated in the paper is that the spatial correlation and heteroskedasticity in the residuals could not be tested in the absence of an appropriate sampling theory. This problem is still open and remains an area of future research.

6. STOCHASTIC CONVERGENCE AND OTHER TIME SERIES APPROACHES

As already pointed out, most of the empirical analysis uses traditionally cross-sectional econometric techniques in testing convergence hypotheses. However, recently in the applied (not necessarily *spatial*) econometric literature on growth and convergence some alternatives were proposed that, departing substantially from the simple β-convergence approach, introduce in one way or the other the time dimension explicitly into discussion. Even if these models so far did not take into account explicitly any spatial effect amongst regions, it is interesting to review some of them here because their framework can be easily adapted in the future to include the spatial dimension.

One of these approaches, that can be considered an important step forward with the respect to the neoclassical growth convergence modelling, is the one based on the concept of stochastic convergence introduced by Bernard and Durlauf (1995, 1996). These authors proposed a new definition of convergence based on the unit-root concept developed in the context of time series analysis (Davidson and MacKinnon, 1993). If technological progress, which drives the long-run economic growth, contains a stochastic trend, then the convergence implies that permanent components in GDP are the same across regions. In this context convergence is presented as a “catching up over a certain time period”.

According to Bernard and Durlauf definition countries *i* and *j* thus convergence if the long-term forecast of output for both countries are equal at a fixed time t:

$$\lim_{t \Rightarrow \infty} E\left(y_{j,t} - y_{i,t} \middle| I_0\right) = 0 \qquad \textbf{[12]}$$

where I_0 represent the information set at time 0.

Stochastic convergence thus occurs if the difference between benchmark real GDP per capita and group country real per capita follows a stationary process.

The well-known univariate augmented Dickey-Fuller (ADF) (see e.g. Davidson and MacKinnon, 1993) given by the following equation:

$$\Delta y_{it} = \alpha_i + \rho_i y_{i,t-1} + \gamma_i t + \sum_{j=1}^{K} \delta_{ij} \Delta y_{i,t-j} + \varepsilon_{it} \qquad \textbf{[13]}$$

(with i = 1,…….., n, t =1,….T and j = 1,……., k ADF lags) can then be used to test the hypothesis of regional convergence. Notice the two different meanings assigned to the term convergence in this context. Notice also that they are both different from the term as it is used in the preceding section when dealing with mathematical convergence. Thus in this paper we reach the third different meaning for the same word "convergence". To the standard concept of "regional convergence" we added a concept of "mathematical convergence" in the sense explained in Section 5 and now the idea of "stochastic convergence". There are, obviously, relationships between these three concepts. However they are not straightforward and they are not analysed any further in the present context.

In the ADF test the null hypothesis is that the pair-wise differences between a region chosen as the benchmark and all other regions follow a unit-root process and therefore the regions do not converge stochastically. Evidences from univariate unit-root test often show that OECD and European GDP differentials persist and economies tend to diverge (see e.g. Flessig and Strauss, 2001). These results, however, were attributed to the low power of univariate unit-root tests and to remove this problem alternative panel unit-root tests were suggested. Amongst these Levin and Lin (1992) formulated a panel unit root test procedure that allows the residual variance and the pattern of higher-order serial correlation to vary freely across individuals. Similarly Im, Pesaran, and Shin (see Im et al., 1997) developed a panel unit root test that allow for heterogeneity in the value of ρ_i under the alternative hypothesis. Using this formulation the term ρ_i in Equation (13) may differ across groups and may display a geographical pattern. In other words, the Im-Pesaran-Shin test evaluates the null hypothesis that all of the series contain unit-roots against the alternative hypothesis that some series are stationary.

Taylor and Sarno (1998) proposed a multivariate ADF test for unit roots that allows for different values of ρ_i . This approach consists in testing the null hypothesis that each series has a unit root ($\rho_i = 0$ for all *i)* against the alternative that at least one series is stationary ($\rho_i < 0$ for some *I)* The use of a Wald test statistics was proposed, that follows χ^2-square distribution with n degrees of freedom under the null hypothesis. However Taylor and Sarno (1998) calculated its finite-sample empirical distribution obtained via Monte Carlo simulation.

In Arbia and Costantini (2004) we adopted a panel unit-root procedures to test stochastic convergence of Italian regions over the period 1951-2002 and we applied the Levin-Lin and Im-Pesaran-Shin testing procedures. We considered the full panel of the 20 Italian regions and two sub-samples refereed to only the Northern regions and, respectively, the Centre-Southern regions. Starting from the agreed fact that convergence has occurred in this long period (see e.g. Paci and Pigliaru, 1997; and Arbia, Basile e Salvatore, 2003), we split the whole period into two sub-periods (1971-1976, 1977-2002) in order to take into account the effects of the first oil crisis occurred in the 1973-1974. We then evaluated the stochastic convergence among Italian "macro-regions" and we analysed the robustness of stochastic convergence hypothesis over the time period considered. The null hypothesis was that regional economies did not converge stochastically or, in other words, that the residuals in Equation (13) contain a unit root. The benchmark region chosen was Lombardia. For all regional economies, univariate ADF test failed to reject the null hypothesis of no stochastic convergence at 5 % significant level with one, two and three lag difference terms. Considering the two sub-periods and the two groups of regions, our findings show a weak evidence of stochastic convergence for the Northern and Centre-Southern and a stronger one in the second sub-period (1976-2002) for all regions. These results highlight the importance of analysing stochastic convergence allowing the geographical dimension to enter the discussion and hence the need for explicit spatial econometric modelling. It is noticeable, however, that some of the concepts in time series analysis pertaining unit roots and cointegration have already been investigated in the context of spatial econometrics (Fingleton, 1999; Mur and Trìvez, 2003). Getis and Griffith (2002), notice that this important topic is still absent in the treatment of other spatial problems with the only noticeable exception of the work by Griffith and Tiefelsdford (2002).

Harvey and Carvalho (2002) are also interested in the dynamics of convergence rather than its occurrence within a certain time period and propose a second-order error correction mechanism embedded within a stochastic convergence framework that provides an informative decomposition into trend, cycle and convergence components. They also show that time series test of whether economies converge can be formulated within this framework, but again do not provide any explicit treatment to treat spatial effects.

Finally within the context of stochastic convergence it is interesting to consider the approach proposed recently by Pesaran (2004a) based on the computation of convergence measures derived considering all possible pairs of (log) per-capita output gaps across n economies no matter what their position is in space.

7. SUMMARY AND CONCLUSIONS

This is a review paper that discusses some of the alternatives proposed in the recent econometric literature to the standard regional convergence modelling strategies. The common feature of the models considered here is the criticism towards the use of purely cross-sectional data with the related neglecting of the time dimension. Conversely an explicit consideration of the full dynamic of the regional economies seems to be crucial when analysing the pattern of convergence. Four main approaches are reviewed here. The more general methodology that has not produced so far any application to the specific case of regional convergence is that of the space-time series analysis. This approach is considered in Section 2 of the paper. A second approach that has indeed already produced some remarkable results in the analysis of per-capita GDP convergence is linked to the developments recorded recently by the panel data econometrics that consider explicitly the spatial dimension. This second approach together with and some empirical results are reviewed in Section 3. A third approach is the one based on space-time modelling, but developed with a continuous time framework. This is developed in Section 4 in a general way that can encompass various spatial effects. Some of these have been already developed in the literature and the empirical results were reviewed in this section. Finally in Section 5 we discussed the possibility of using the idea of stochastic convergence within the context of a spatial econometric approach. Here the methodologies seem to be at an early stage of developments and the empirical analysis so far have been confined to only the time dimension. However the approach seems to be promising of bearing fruits in the future.

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